SovietRxiv

Soviet-era science, translated into English

APPROXIMATE SOLUTION OF PROBLEMS IN THE THEORY OF SMALL ELASTO-PLASTIC DEFORMATIONS

1957

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-195701.86681

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

THEORY OF ELASTICITY

D. D. IVLEV

APPROXIMATE SOLUTION OF PROBLEMS IN THE THEORY OF SMALL ELASTO-PLASTIC DEFORMATIONS

(Presented by Academician A. I. Nekrasov, 12 X 1956)

We shall seek the solution of the problem in the form of series in a certain parameter δ:

$$\sigma_\rho = \sum_{n=0} \delta^n \sigma_\rho^{(n)}, \dots, \quad u = \sum_{n=0} \delta^n u^{(n)}, \dots, \quad e_\rho = \sum_{n=0} \delta^n e_\rho^{(n)}, \dots \tag{1}$$

If for $\delta = 0$ an axisymmetric stressed state occurs, then, neglecting compressibility, in the case of plane deformation we shall have

$$u^0 = -\frac{c}{\rho}, \quad e_\rho^0 = -e_\theta^0 = \frac{c}{\rho^2}, \quad e_i^0 = \frac{2}{\sqrt{3}} \frac{c}{\rho^2}, \quad c = \text{const.} \tag{2}$$

Substituting the expansions (1) into the relations connecting stresses and deformations in the theory of small elasto-plastic deformations (1):

$$\sigma_\rho - \sigma_\theta = \frac{4}{3} \frac{\sigma_i}{e_i} e_\rho, \quad \tau_{\rho\theta} = \frac{1}{3} \frac{\sigma_i}{e_i} e_{\rho\theta},$$

and assuming that $\sigma_i = A e_i^\chi$, using (2), we obtain:

$$\sigma_\rho^{(n)} - \sigma_\theta^{(n)} = 4B\chi\rho^p e_\rho^{(n)} + F_n, \quad \tau_{\rho\theta}^{(n)} = B\rho^p e_{\rho\theta}^{(n)} + \Phi_n, \tag{3}$$

where

$$B = \frac{A}{3} \left(\frac{2c}{\sqrt{3}} \right)^{\chi - 1}, \quad p = 2(1 - \chi),$$

and the functions F_n and Φ_n depend on components not higher than the $(n-1)$-st approximation.

Assuming that the $(n-1)$-st approximation has been determined, we determine the n-th approximation. Put

SovietRxiv

Fig. 1

Figure 1: Fig. 1

$$u^{(n)} = -\frac{1}{\rho}\frac{\partial \Psi_n}{\partial \theta}, \quad v^{(n)} = \frac{\partial \Psi_n}{\partial \rho}, \quad \Psi_n = \rho R(\rho)\Theta(\theta). \tag{4}$$

From (3) and (4) we have

$$\sigma_\rho^{(n)} - \sigma_\theta^{(n)} = -4B\chi\rho^p R'\dot{\Theta} + F_n,$$

$$\tau_{\rho\theta}^{(n)} = B\rho^p \left[\left(\rho R'' + R' - \frac{R}{\rho} \right) \Theta - \frac{R}{\rho}\ddot{\Theta} \right] + \Phi_n, \tag{5}$$

where a prime above denotes differentiation with respect to ρ, and a dot denotes differentiation with respect to θ.

Setting $\Theta = \cos m\theta$ or $\sin m\theta$, from (5) and the equilibrium equations we obtain:

$$\rho^4 R^{\mathrm{IV}} + 2(p+3)\rho^3 R''' + [p^2 + 6p + 5 + 2m^2(1-2\chi)]\rho^2 R''$$
$$+\rho R'[(p^2-1) - 2m^2(2\chi-1)(p+1)] + R[m^4 + (1-p^2) - (2-p^2)m^2] = U_n, \tag{6}$$

where the right-hand side of equation (6) is a known function of the radius ρ.

We find the general solution of the homogeneous equation (6). Setting $R = \rho^k$, we obtain from (6):

$$k^4 + 2pk^3 + \left[p^2 - 2 - 2(2\varkappa - 1)m^2\right]k^2 - 2p\left[1 + (2\varkappa-1)m^2\right]k+$$
$$+\left[(1-p^2) - (2-p^2)m^2 + m^4\right] = 0. \tag{7}$$

Fig. 1

Equation (7) admits a factorization:

$$(k^2 + pk + a + ib)(k^2 + pk + a - ib) = 0, \tag{8}$$

where

$$a = -\left[1 + (2\varkappa - 1)m^2\right], \qquad a^2 + b^2 = (1-m^2)^2 + p^2(m^2-1);$$

$$b^2 = 4\left\{m^4\varkappa(1-\varkappa) + m^2\left[(1-\varkappa)^2 - \varkappa\right] - (1-\varkappa)^2\right\}.$$

SovietRxiv

The factorization of equation (7) into factors (8) may occur under certain relations between $\varkappa$ and m. In Fig. 1 the graph for $b = 0$ is presented; the factorization (8) takes place in the zone of complex roots of equation (7). The roots of equation (8) are

$$k_{1,2,3,4} = \frac{1}{2}\left\{-p \pm \sqrt{\frac{1}{2}\left[\sqrt{(p^2-4a)^2+16b^2}+(p^2-4a)\right]}\right.$$
$$\left.\pm i\sqrt{\frac{1}{2}\left[\sqrt{(p^2-4a)^2+16b^2}-(p^2-4a)\right]}\right\}. \qquad (9)$$

For $m = 1$, equation (7) takes the form

$$k\left[k^3 + 2pk^2 + \left[p^2 - 2 - 2(2\varkappa - 1)\right]k - 2p\left[1 + (2\varkappa - 1)\right]\right] = 0. \qquad (10)$$

The roots of equation (10) (see also [2]) are

$$k_1 = 0, \qquad k_2 = -2(1-\varkappa), \qquad k_3 = 2\varkappa, \qquad k_4 = -2. \qquad (11)$$

Using the values of the roots (9) and (11), it is easy to write down the general solution of the homogeneous equation (7). Finding a particular solution of equation (7) in concrete problems presents no difficulty. Using the general solution of equation (7), one can obtain solutions of problems on an eccentric tube under the action of external and internal pressures, on an elliptical tube, on biaxial stretching of a thick plate with a circular or elliptical hole, etc.

We note that, analogously, we arrive at equation (7) if for $\delta = 0$ symmetric torsion takes place:

$$\sigma^0_\rho = \sigma^0_\theta = 0, \qquad \tau^0_{\rho\theta} = \frac{\tau_0}{\rho^2}, \qquad \rho \geqslant \alpha \neq 0.$$

Moscow State University
named after M. V. Lomonosov

Received
10 X 1956

References Cited

[1] A. A. Ilyushin, *Plasticity*, 1948. [2] L. M. Kachanov, *Izv. AN SSSR, OTN*, no. 9 (1956).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.