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Abstract

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THE PROFILE OF WIND WAVES AND ITS RECORDING ON BOTTOM-MOUNTED WAVE RECORDERS

Extensive investigations of ocean and sea waves, organized in all countries during the Third International Geophysical Year, will rely to a considerable extent on records from wave recorders with bottom-mounted pressure sensors. Meanwhile, it is well known that the curves obtained on such wave recorders, as a rule, differ sharply from the true profile of wind waves. The qualitative explanation of such a difference is obvious: as one moves away from the sea surface toward the bottom, not only do the full amplitudes of the pressure oscillations continuously decrease, but at the same time the overtones of various orders, inherent in the real profile of strongly sharpened wind waves, are as it were "filtered out."

In the present article an attempt is made at a quantitative investigation of such "filtering," proceeding from our present-day kinematic ideas about the wind wave (1). To begin with, let us recall that the classical law of pressure oscillations p at depth y during the passage of waves,

$$\frac{p}{\delta} = -\frac{\partial \Phi}{\partial t} - \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2\right] - gy \tag{1}$$

(where δ is the density of sea water; t is time; g is the acceleration in the field of gravity), is obtained in its simplest form if the velocity potential is expressed by the single-term formula

$$\Phi = F \operatorname{ch} k(y+H)\cos(kx - \omega t),$$

$$k = \frac{2\pi}{\lambda}, \qquad \omega = \frac{2\pi}{T}, \tag{2}$$

i.e., if Φ varies according to a simple cosine law. Specifically, it is easy to show that the difference Δp between the extreme values of the bottom pressure at depth $y = -H$ during the passage of such schematized simple waves is expressed as

Fig. 1

Figure 1: Fig. 1

$$\frac{\Delta p}{\delta g} = h\left[\frac{1}{\mathrm{ch}(2\pi H/\lambda)} + \frac{\pi}{2}\frac{h/\lambda}{\mathrm{sh}(4\pi H/\lambda)}\right]. \tag{3}$$

The quantities δ and g have been transferred to the left-hand side so that this side is expressed in the same linear units in which the wave height h is measured. For ordinary values of h/λ and H/λ, the second term on the right-hand side may be neglected. In that case (3) gives the functional dependence of η on H/λ, graphically represented in Fig. 1, where η is equal to the ratio of the difference of the extreme values of the bottom pressure in millimeters of water column to the wave height in meters:

$$\eta = \frac{1000\Delta p/\delta g}{h}.$$

However, formula (3) and the diagram in Fig. 1 are practically applicable only to waves of gently sloping swell. Sharpened wind waves require taking into account many terms of the Fourier series, in addition to the first one included in formula (2).

At first glance, the approximate equation for the profile of steep waves obtained in one of Rayleigh' s papers ([2]) might seem sufficient:

$$y = -\frac{1}{2}\alpha^2 - \alpha^4 + \left[\alpha + \frac{9}{8}\alpha^3 + \frac{769}{192}\alpha^5\right]\cos x - \left[\frac{1}{2}\alpha^2 + \frac{11}{6}\alpha^4\right]\cos 2x + \left[\frac{3}{8}\alpha^3 + \frac{315}{128}\alpha^5\right]\cos 3x - \frac{1}{3}\alpha^4\cos 4x + \frac{125}{384}\alpha^5 c \tag{4}$$

Here the parameter α and the argument x are dimensionless quantities chosen so that $\alpha = \pi h/\lambda$; $c^2 = g\lambda/2\pi = 1$. It is thereby predetermined that the coefficients of the Fourier series in (4), beginning with a_0, expressed by the two free terms of the right-hand side, up to a_5, expressed through the fifth power of α, are connected only with the ratio of the wave height h to the length λ, but are not connected directly with the wave height. Such a connection would be found if the series on the right-hand side of (4) converged very rapidly. Meanwhile, analysis of actual profiles of wind waves shows that, in any case beginning with a_4, the coefficients of the cosines decrease much more slowly than according to Rayleigh' s theory. It is precisely for this reason that the coefficient a_1 in nature constitutes a noticeably smaller fraction of the total height of pointed wind waves than it should according to (4).

Fig. 1

Fig. 2

Figure 2: Fig. 2

For the investigation of natural profiles of wind waves, the parameter α entering into (4) is also unsuitable; as we have been able to show ([1,3]), the pointedness of waves depends not only on the ratio h/λ, but also on the ratio of the mean velocity of the pulsating drift current $\bar{u}$ to the velocity of the orbital motion v of a surface water particle on the wave. If the motion of the surface particle is referred to a coordinate system moving in the direction of the wave with a velocity equal to the sum of $\bar{u}$ and $\bar{w}$ (where $\bar{w}$ is the mean velocity of the so-called Stokes wave current), then relative to this coordinate system the particle will describe an ellipse with semiaxes a and b, the ratio of which increases with increasing $\bar{u}/v$ and r/R (where r is the half-height of the wave, and R is the radius of the circle of rolling). Together with it the pointedness of the waves increases. In accordance with our works ([1,3]), we shall characterize the pointedness of waves by the ratio of the segments AB/BC (see Fig. 2). Figure 2 gives a family of profiles of wind waves corresponding to different values of $\chi = AB/BC$. The oscillations of the sea level are given as a function of the current time t and occur with a common period T. The curves were obtained from profiles calculated according to our kinematic equations

$$x = R\theta + a\sin\theta, \qquad y = b\cos\theta. \tag{5}$$

Fig. 2

The profile corresponding to $\chi = 0.38$ gives an extremely steep wave, which in complete calm would be characterized by the ratio $h/\lambda = 1/7$. With an increase in the wind speed and the associated increase in the ra-

of the ratio u/v, the limiting steepness is reached already at $h/\lambda < 1/7$. The profile corresponding to $x = 0.9$ approaches a simple sinusoidal one.

Analysis of the curves in Fig. 2, carried out by means of a Mader–Ott harmonic analyzer, showed that the coefficients a_n of the Fourier series vary as a function of the argument x in the manner shown in Fig. 3. The Roman numerals by the curves indicate the numbers of the harmonics n. The heights of waves of the n-th order are expressed as fractions of the total wave height h. As we see, the height of the fundamental waves is 0.81 of the total height of the composite waves in the case of their greatest sharpening ($x = 0.38$).

On the basis of well-known trigonometric relations it can be shown that, generally speaking,

$$h/2 = a_1 + a_3 + a_5 + \cdots,$$

Fig. 3

Figure 3: Fig. 3

i.e., the total height of composite waves depends only on the odd harmonics. On the other hand, only on the even harmonics depends the difference between the segments AB and BC for all composite curves; in other words, the argument x depends only on the even harmonics.

As one moves away from the sea surface toward the bottom, the n-th harmonics must decrease in amplitude considerably faster than the amplitude of the fundamental oscillation decreases according to equation (3). As a result, the curve of variation of bottom pressure with time must lose practically all, or almost all, harmonics inherent in the profile of sharpened surface waves. As a rule, this curve, recorded by means of a bottom wave recorder, is very close to a sinusoid. Meanwhile, on the basis of (3), by the external sign—the presence of foamy crests ("whitecaps")—one can sometimes conclude without error that the profile of wind waves is maximally sharpened ($x = 0.38$).

Fig. 3

Under such conditions it should be taken into account that a wave recorder with a bottom pressure sensor records not simply a distorted profile, but also gives an underestimated value of the total height, if it is recalculated by formula (3) or by the diagram in Fig. 1: the amplitude of the fundamental oscillations here amounts to only 0.81 of the half-height of the composite waves.

Some foreign authors have empirically compared records of bottom wave recorders with the actual dimensions of waves and have sometimes found even more underestimated results: values amounting to about 0.78 of that calculated by (3). In the opinion of S. V. Dobroklonsky, the influence of bottom friction was involved here, which is not taken into account in our calculations; this should be checked in experiments in a storm basin.

In this connection it is interesting to compare the ratios of the various coefficients a_n to the first coefficient a_1, first, as calculated from the short Rayleigh series (4), and, second, as determined by harmonic analysis of our family of curves (5) (Fig. 2). We shall make the comparison for the case of complete calm, to which Rayleigh' s theory applies.

Taking into account that Rayleigh' s parameter α is identically equal to the quantity r/R appearing in our formulas, we shall write, instead of relations (70) and (92) of work (3),

$$\frac{a}{b} = \frac{\pi}{2\alpha}\left(\frac{1-x}{1+x}\right); \qquad \frac{a}{b} = 1 + \alpha. \tag{6}$$

Equating both expressions for a/b in (6) and solving the resulting equation with respect to α, we find

$$2\alpha = -1 + \sqrt{1 + 2\pi\left(\frac{1-\chi}{1+\chi}\right)}. \quad (7)$$

Determining from (7) the values of α for 6 values of χ in Fig. 2 and calculating the ratios $a_2/a_1,\ a_3/a_1,\ a_4/a_1,\ a_5/a_1$ by (4), we obtain Table 1.

Table 1

χ	Method	a_2/a_1	a_3/a_1	a_4/a_1	a_5/a_1
0.38	By Rayleigh	0.281	0.126	0.0217	0.0096
0.38	By the new kinematics	0.289	0.123	0.074	0.0444
0.50	By Rayleigh	0.234	0.085	0.0148	0.0054
0.50	By the new kinematics	0.252	0.090	0.036	0.0225
0.60	By Rayleigh	0.176	0.0493	0.00795	0.0023
0.60	By the new kinematics	0.175	0.0486	0.0296	0.017
0.70	By Rayleigh	0.125	0.0253	0.0037	0.0008
0.70	By the new kinematics	0.127	0.0331	0.0103	0.0062
0.80	By Rayleigh	0.0788	0.0095	0.0011	0.0002
0.80	By the new kinematics	0.0830	0.0182	0.0101	0.0081
0.90	By Rayleigh	0.0430	0.00263	0.0002	0.00012

χ	Method	a_2/a_1	a_3/a_1	a_4/a_1	a_5/a_1
0.90	By the new kinematics	0.0405	0.00	0.00	0.00

This table makes it possible to draw the following conclusions:

1. A comparison of the quantities a_2/a_1, obtained by the two methods, for all values of the argument χ, again confirms the correctness of the fundamental propositions of our kinematics of peaked wind waves.

2. In fact, as applied to these waves, the Fourier series converges more slowly than according to Rayleigh' s theory.

3. Harmonic analysis of the profiles obtained from formula (5) makes it possible directly to determine the values $\frac{a_n}{h/2}$, which cannot be determined directly from Rayleigh' s theory.

4. The empirical conclusions of foreign authors are closer to the theoretical conclusions of the present article than to the conclusions from Rayleigh' s theory. Thus, for the case of the most peaked waves ($\chi = 0.38$), some foreign empirical determinations gave $\frac{a_1}{h/2} = 0.78$; our theory gave $\frac{a_1}{h/2} = 0.81$; Rayleigh' s theory, instead of the quantity $\frac{a_1}{h/2}$ itself, gives the approximate quantity: $\frac{a_1}{a_1 + a_3 + a_5}$, which in the present case is equal to $1/(1 + 0.126 + 0.0097) = 0.883$.

5. For practical purposes one may adopt the diagrams in Figs. 2 and 3, by which the actual height of complex peaked waves is approximately determined on the basis of the distorted record of wave gauges with a bottom pressure sensor.

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