# On the Radial Displacement of Gas-Saturated Oil by Water

1957

SovietRxiv

## Abstract

## Full Text

## Hydromechanics

**M. D. Rozenberg**

# On the Radial Displacement of Gas-Saturated Oil by Water

*(Presented by Academician A. I. Nekrasov, 4 IX 1956)*

In our joint work with M. M. Glogovskii ([3]), an approximate solution was considered for the problem of the radial displacement of gas-saturated oil by edge water. In particular, the case was considered of radial displacement of oil by water while maintaining a constant pressure at the supply contour of the water-bearing region.

Analysis of the solution obtained in that work led to the conclusion that, after a certain comparatively short period, gas-saturated oil displaced by water moves practically as an incompressible liquid, and made it possible to propose an engineering method for calculating the displacement of gas-saturated oil by water ([4]). It should be noted that in this solution some real properties of reservoir oils and gases were not taken into account: the dependence of the viscosities of oil and gas on pressure, deviations from the laws for ideal gases, and the change in the oil formation volume factor during degassing. In the present work a more general problem of the radial displacement of gas-saturated oil by water is considered.

Consider a circular reservoir (Fig. 1) of radius $r_k$, at the center of which, at the instant $t = 0$, a well of radius $r_c$ begins to operate. We shall assume that in the water-bearing region, at a distance $r_0$ from the center of the reservoir, either a constant pressure or the rate of water invading the formation is prescribed.

*Fig. 1*

In the water-bearing region, neglecting the elasticity of water, we shall regard the regime as gravitational-water-drive. Suppose that the saturation pressure is equal to the initial reservoir pressure. We shall also assume that the oil is completely displaced by water.

Introduce the notation: $\tilde{p}$ is the average pressure in the oil reservoir; $\tilde{\rho}$ is the average saturation of the pores with oil in the oil region; $\Omega_k$ is the volume of pore space in the oil region, where

$$\Omega_k = \pi h m \left(r_k^2 - r_c^2\right); \qquad (1)$$

$\beta(p)$ is the oil formation volume factor.

We note that over almost the entire interval of pressure variation and, in any case, over the entire interval of practical interest for the case under study, $\beta(p)$ is a linear function of pressure ([5]). Then the oil production rate will be expressed in the following form:

$$q_n = -\frac{d}{dt}\left[\frac{\tilde{\rho}\Omega_k}{\beta(\tilde{p})}\right], \tag{2}$$

and the rate of water that has invaded the reservoir will be equal to

$$q_{\mathrm{w}} = -\frac{d\Omega_k}{dt}. \tag{3}$$

Dividing (2) by (3) and taking (1) into account, after simple transformations we obtain:

$$\frac{d\left[\dfrac{\tilde{\rho}}{\beta(\tilde{p})}\right]}{dr_k} = \frac{2}{r_k}\left[\frac{q_{\mathrm{o}}}{q_{\mathrm{w}}} - \frac{\rho}{\beta(\tilde{p})}\right]. \tag{4}$$

Putting $\beta(p) = 1$ in equation (4), we obtain the equation considered in work ([3]). Differential equation (4) makes it possible to solve the problem of the radial displacement of gas-saturated oil by water under various prescribed conditions. On the boundary of supply of the water-bearing region, either the values of the instantaneous rate of water invading the reservoir or the pressure may be specified, and at the well—the oil production rate or the bottom-hole pressure. To each type of such conditions there will correspond one of the special cases of equation (4).

Let us note that in the simplest case, when a constant water rate and oil production rate, or their constant ratio, are specified, equation (4) is easily integrated.

Of greatest practical interest is the case when a constant pressure is specified on the supply boundary. We shall assume that it is equal to the initial reservoir pressure—$p_0$. In what follows we shall regard the unsteady motion as a succession of steady states ([1,2]). We introduce the gas factor, defined by the equality:

$$\Gamma = \frac{\gamma(p)}{\gamma_0}\frac{\mu_{\mathrm{o}}(p)}{\mu_{\mathrm{g}}(p)}\psi(p)\beta(p) + \frac{S(p)}{\gamma_0}, \tag{5}$$

where $\gamma$ is the specific weight of free gas, $\mu_{\mathrm{o}}(p)$ and $\mu_{\mathrm{g}}(p)$ are the viscosities of oil and gas, $S(p)$ is the weight of gas per unit of solution, and $\psi(p)$ is the ratio of the phase permeabilities. The gas production rate is

$$q_{\mathrm{g}} = -\frac{d}{dt}\left[\Omega_k(1-\tilde{\rho})\frac{\gamma_{\mathrm{g}}(\tilde{p})}{\gamma_0}\right] - \frac{d}{dt}\left[\Omega_k\tilde{\rho}\frac{S(\tilde{p})}{\beta(\tilde{p})\gamma_0}\right]. \tag{6}$$

The pressure functions on the right-hand side of (6) are assumed to be linear (which, for our case of constant pressure at the boundary, is a sufficiently good approximation).

From equations (2), (4), and (6), taking (1) and (5) into account, after transformations we obtain:

$$\frac{d\tilde{p}}{dr_k} = \frac{2}{r_k} f(\tilde{\rho}, \tilde{p}, r_k),$$

$$\frac{d\tilde{\rho}}{dr_k} = \frac{2\beta(\tilde{p})}{r_k} \left[ \frac{q_\mathrm{o}\mu_\mathrm{w} \ln\left(\frac{r_0}{r_k}\right)}{2\pi k h (p_0 - \tilde{p})} - \frac{\tilde{\rho}}{\beta(\tilde{p})} \right] - \frac{2\tilde{\rho}}{r_k} \frac{\beta'(\tilde{p})}{\beta(\tilde{p})} f(\tilde{\rho}, \tilde{p}, r_k), \tag{7}$$

where

$$f(\tilde{\rho}, \tilde{p}, r_k) = \tag{8}$$

$$= \frac{(1-\tilde{\rho})\frac{\gamma_\mathrm{g}(\tilde{p})}{\gamma_0} + \tilde{\rho}\frac{S(\tilde{p})}{\beta(\tilde{p})\gamma_0} - \Gamma \frac{q_\mathrm{o}\mu_\mathrm{w} \ln\left(\frac{r_0}{r_k}\right)}{2\pi k h (p_0 - \tilde{p})} - \left[ \frac{q_\mathrm{o}\mu_\mathrm{w} \ln\left(\frac{r_0}{r_k}\right)}{2\pi k h (p_0 - \tilde{p})} - \frac{\tilde{\rho}}{\beta(\tilde{p})} \right] \left[ \frac{\gamma_\mathrm{g}(\tilde{p})}{\gamma_0} - \frac{S(\tilde{p})}{\gamma_0} \right]}{(\rho - 1)\frac{\gamma'(\tilde{p})}{\gamma_0} + \frac{\tilde{\rho}}{\beta(\tilde{p})}\frac{\gamma_\mathrm{g}(\tilde{p})}{\gamma_0}\beta'(\tilde{p}) - \frac{\tilde{\rho}}{\beta(\tilde{p})}\frac{S'(\tilde{p})}{\gamma_0}}.$$

System (7) is integrated numerically. As initial conditions one may take the values of the average pressure and average saturation calculated by the method of changing steady states up to the moment when the contour of the disturbed region reaches the oil-bearing contour. If, however, the first phase of filtration of the gassy liquid is neglected, which is permissible in view of its insignificant duration in comparison with the total time of development of the reservoir, then the initial conditions will be

$$\tilde{\rho} = \rho_0, \qquad \tilde{p} = p_0.$$

In this case system (7) has a singularity, and the integration of the equations should be begun after transforming (7) to the independent variable $\tilde{\rho}$. Having found the integral of the equations in the neighborhood of the singularity, we return to system (7) with the independent variable $r_k$, since only it is connected with time by a one-to-one dependence. Having integrated (7) numerically, we find the dependence of $\tilde{p}$ and $\tilde{\rho}$ on $r_k$.

Fig. 2

Figure 1: Fig. 2

Fig. 2

Let us consider in detail the case of a prescribed constant withdrawal of oil from the reservoir. In this case $q$ in equations (7) and (8) will be constant. The relation between $r_k$ and time for this case is found from equation (3) in the form:

$$t = \frac{\pi m h}{q}\left[\frac{r_{k_0}^2 \tilde{\rho}_0}{\beta(p_0)} - \frac{r_k \tilde{\rho}}{\beta(\tilde{p})}\right]. \tag{9}$$

The quantity of water invading the reservoir can be calculated from the formula:

$$q = \frac{2\pi k h\,(p_0 - \tilde{p})}{\mu \, \ln\left(\dfrac{r_0}{r_k}\right)}. \tag{10}$$

The gas factor is determined from (5). The pressure at the bottomhole of the oil well can be found from the prescribed oil rate and the calculated values $r_k$, $\tilde{\rho}$, $\tilde{p}$ with the aid of the Khristianovich functions ([1,5]).

This method was used to investigate a case with the following initial data: $r_0 =$ 3000 m; $r_{k_0} = 500$ m; $p_0 = 76$ atm; $q = 150$ m$^3$/day; $k = 1$ darcy; viscosity of the oil at a pressure equal to the saturation pressure, 2 cP; gas viscosity 0.012 cP; $\gamma\,(\tilde{p}) =$ const. The real properties of the oil and gas were taken from works ([6,7]).

Figure 2 presents the results of the calculations. As can be seen, the process of displacement of gassy oil by water can be divided into two periods.

In the first of them, the pressure and saturation decrease sharply, just as in the case of the dissolved-gas drive regime. At the same time the water flow rate rapidly

increases from zero to a value equal to the total withdrawal of oil and gas from the reservoir under reservoir conditions.

At the beginning of the second period, the amount of water invading the reservoir exceeds the withdrawal of oil and gas, after which a rapid increase in saturation and an insignificant increase in pressure are observed. Further, the rate of water invading the reservoir, after passing through a maximum, approaches the total withdrawal of oil and gas from the reservoir. The saturation of the pore space with oil in the second period first increases sharply and then, having reached a certain value, changes almost no further. The gas factor, having reached a maximum at the end of the first period, at the beginning of the second period rapidly reaches its initial value, and then slowly decreases. The pressure $\tilde{p}$ at the very beginning of the second period increases somewhat, after which it begins to decrease, in the same way as occurs when incompressible oil is displaced by water.

**Table 1**

| $t$ (days) | $\Delta r_k$, m: gas-saturated liquid | $\Delta r_k$, m: incompressible liquid | Deviation, % |
|---|---|---|---|
| 3.13 | 0.04 | 1.03 | 2475 |
| 14 | 0.77 | 4.51 | 486 |
| 52.06 | 8 | 16.24 | 103 |
| 103.46 | 24 | 31.79 | 32 |
| 206.59 | 60 | 63.63 | 6 |
| 307.92 | 95 | 97.44 | 2.5 |
| 415.99 | 135 | 137.02 | 1.4 |
| 609.32 | 220 | 221.49 | 0.6 |
| 806.90 | 350 | 350.77 | 0.2 |

It should be noted that the increase in saturation at the beginning of the second period is explained, as is evident from the behavior of the pressure, not by the reverse dissolution of gas in the oil, but by the displacement of part of the gas that has separated from solution.

As was already noted above, the saturation $\tilde{\rho}$ is not a one-to-one function of time. This circumstance is clearly seen from Fig. 2. Therefore, the choice of $\tilde{\rho}$ as the independent variable in solving the problem, as was done in ([8]), where the particular case of the motion of an idealized liquid was considered, is not justified and does not make it possible to obtain the complete solution of the problem.

Analysis of the solution obtained in the present work shows that, with time, the quantity

$$\frac{q}{q} - \frac{\tilde{\rho}}{\beta(\tilde{p})}$$

tends to zero, and the process of displacement of gas-saturated oil by water approaches the process of displacement of dead oil by water.

Table 1 gives a comparison of the advance of the oil-bearing contours when gas-saturated oil and incompressible oil are displaced by water, for equal withdrawal of liquid under reservoir conditions.

As is seen from Table 1, during the main period gas-saturated oil is displaced by water practically as an incompressible liquid. Thus, the principal conclusion obtained earlier from a simplified analysis ([3,4]) is confirmed.

The character of the approximation of the process of displacement of gas-saturated oil by water to the process of displacement of an incompressible liquid is somewhat different than according to the results of the works ([3,4]).

All-Union Oil and Gas
Scientific Research Institute

Received
19 XI 1955

## References

1. S. A. Khristianovich, *Prikl. matem. i mekh.*, **5**, issue 2 (1941).
2. K. A. Tsarevich, *Tr. MNI*, issue 5 (1947).
3. M. M. Glogovskii, M. D. Rozenberg, *DAN*, **85**, No. 6 (1952).
4. M. M. Glogovskii, M. D. Rozenberg, *Tr. MNI*, issue 12 (1953).
5. L. A. Zinov' eva, *Tr. MNI*, issue 6 (1954).
6. A. Yu. Namiot, M. A. Regel' man, G. F. Trebin, *Tr. Vsesoyuzn. neftegaz. nauch.-issl. inst.*, No. 2 (1952).
7. G. F. Trebin, A. Yu. Namiot, *Tr. Vsesoyuzn. neftegaz. nauch.-issl. inst.*, No. 2 (1952).
8. G. P. Guseinov, *Dokl. AN AzerbSSR*, **10**, No. 2 (1954).

*Note: Figure translations are in progress. See original paper for figures.*