# ON A MECHANICAL TRANSFORMER

!\[Fig. 1 and Fig. 2\](image)

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Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

## Abstract

## Full Text

## MECHANICS

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# ON A MECHANICAL TRANSFORMER

The present paper sets forth the theory of a motion transformer of a new type. The transformer is a kinematic chain shown in Fig. 1. Link 2 rotates about the center $A$ and is a two-sided crank lever with a rigid angle $kAm$ equal to 90°. The sides $Ak$ and $Am$ of lever 2 slide in sliders 5 and 7, which form rotational pairs at points $C$ and $D$ with sliders 4 and 6. Sliders 4 and 6 slide along the crossbar $t$—$t$, belonging to link 3, which moves translationally in the fixed guides $B$.

**Fig. 1** **Fig. 2**

It is not difficult to see that this kinematic chain has two degrees of freedom. Consequently, if point $C$ is guided along some curve $q$—$q$, defined by the equation $\Psi(\xi,\eta) = 0$, then point $D$ will move along the curve $p$—$p$, defined by the equation $\Phi(x, y) = 0$. Thus, the indicated kinematic chain transforms the curve $\Psi(\xi,\eta) = 0$ into the curve $\Phi(x, y) = 0$.

Let us consider the relation between the coordinates of the curves $\Psi(\xi,\eta) = 0$ and $\Phi(x, y) = 0$.

From Fig. 1 it follows directly that

$$x = \xi, \tag{1}$$

$$y = \frac{\xi^2}{\eta}. \tag{2}$$

It is not difficult to see that the chain has the property of reversibility of motion transformation, i.e., if point $D$ is guided along the curve $p$—$p$, then point $C$ will describe the curve $q$—$q$.

Let the curve $\Psi(\xi,\eta) = 0$ be the straight line $q - q$ (Fig. 2), whose equation is

$$\eta = \xi \operatorname{tg}\alpha + n, \tag{3}$$

Fig. 3

Figure 2: Fig. 3

where $\alpha$ is the angle formed by the axis $Cq$ of the fixed guide $E$, in which link 4 moves, with the axis $Ax$. Eliminating the coordinates $\xi$ and $\eta$ from equations (1), (2), and (3), we obtain the equation of the curve $\Phi(x, y) = 0$:

$$x^2 - \operatorname{tg}\alpha \cdot xy - ny = 0, \tag{4}$$

i.e., the equation of an equilateral hyperbola. Thus, the mechanism shown in Fig. 2 is a hyperbolograph.

**Fig. 3**

If the straight line $q - q$ is parallel to the axis $Ax$, then the six-link mechanism shown in Fig. 3 is obtained. We take the coordinate $\eta$ equal to $\eta = 2p$, where $p$ is a certain constant. From Fig. 3 and equations (1) and (2) it follows that

$$x^2 = 2py, \tag{5}$$

i.e., the curve $p - p$ will be a parabola with parameter equal to $p$. It is not difficult to see that this case leads us to Antonov' s parabolagraph, the theory of which was given by N. B. Delone (1).

Let the curve $\Psi(\xi, \eta) = 0$ be a conic section given by an equation of the general form

$$A\xi^2 + 2B\xi\eta + C\eta^2 + 2D\xi + 2E\eta + F = 0. \tag{6}$$

Eliminating the coordinates $\xi$ and $\eta$ from equations (1), (2), and (6), we obtain the equation of the curve $\Phi(x, y) = 0$ in the form

$$Ax^2y^2 + 2Bx^3y + Cx^4 + 2Dxy^2 + +2Ex^2y + Fy^2 = 0. \tag{7}$$

This is a curve of the 4th order; consequently, the kinematic chain shown in Fig. 1 transforms curves of the 2nd order into curves of the 4th order.

If the conic section $\Psi(\xi, \eta) = 0$ is an ellipse with center at point $A$ (Fig. 4) and semiaxes $a$ and $b$,

$$b^2\xi^2 + a^2\eta^2 - a^2b^2 = 0, \tag{8}$$

then the equation of the transformed curve $\Phi(x, y) = 0$ has the form

Fig. 4

Figure 3: Fig. 4

$$y^2 = \frac{a^2}{b^2}\frac{x^4}{a^2 - x^2}. \tag{9}$$

**Fig. 4**

The mechanism for transforming the ellipse (8) into the curve (9) is shown in Fig. 4. Point $C$ of the transformer belongs to link 8, which enters into revolute pairs with sliders 9 and 10, sliding along the fixed ···

axes $Ax$ and $Ay$. In order that the point $C$ of link 8 move along an ellipse with semiaxes $a$ and $b$, it must divide the segment $EF$ in the ratio $EC/CF = a/b$.

If the point $C$ (Fig. 4) moves along the circle

$$\xi^2 + \eta^2 - r^2 = 0, \tag{10}$$

then equation (7) assumes the form

$${}^2y = \frac{x^4}{r^2 - x^2}. \tag{11}$$

This is the "kappa curve." The mechanism for generating the "kappa curve" must have a crank $AC$ (Fig. 4) of constant length $r$. To obtain this mechanism in the mechanism shown in Fig. 4, the links 5, 8, 9, and 10, shown by dashed lines, must be omitted.

If the curve $\Psi(\xi, \eta) = 0$ is a parabola with vertex at the origin,

$$\eta^2 = 2p\xi, \tag{12}$$

then the equation of the curve $\Phi(x, y) = 0$ will be

$$y^2 = \frac{1}{2p}x^3. \tag{13}$$

This is the equation of a semicubical parabola. To obtain, by means of the converter under consideration (Fig. 1), a mechanism for generating a semicubical parabola, one may use the Antonov parabolograph considered above (Fig. 3). The point $D$ of this mechanism traces the parabola $p$—$p$, given by equation (5). Let us attach to the mechanism a two-slider group, shown by dashed lines and consisting of sliders 6 and 7, which enter the revolute pair $E$. Slider 6 must slide along a guide belonging to link 5, and slider 7 along the guide $k$—$k$, belonging to link 2. The guide $Dl$ forms a right angle with the straight line $Ct$. The point $E$ of the mechanism will describe the curve $s$—$s$, whose equation is

$$x'^2 = \frac{1}{2p} y'^3, \tag{14}$$

i.e., a semicubical parabola.

Thus, using the proposed mechanical converter, by adjusting its links one can obtain mechanisms for generating a number of curves of different orders.

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## REFERENCES

1. N. B. Delone, *Transmission of rotation and mechanical tracing of curves by hinge-lever mechanisms*, St. Petersburg, 1894.

*Note: Figure translations are in progress. See original paper for figures.*