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Abstract

Full Text

PHYSICS

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DISTRIBUTION FUNCTIONS FOR A CLASSICAL ELECTRON GAS

(Presented by Academician N. N. Bogolyubov on 27 XII 1957)

For gases in statistical equilibrium, the calculation of the binary distribution function, or, as it is also called, the radial distribution function, is of great interest. Effective methods for constructing such a function were proposed by N. N. Bogolyubov ⁽¹⁾ and are well known.

In the case of Coulomb interaction between particles, expansion in the small parameter v/r_d^3 , where v is the volume per particle; r_d is the Debye radius ($1/r_d^2 = 4\pi e^2/v\theta$; e is the charge of the particle; $\theta = kT$; T is the absolute temperature), gave in ⁽¹⁾ the Debye expression for the radial function:

$$G(r) = 1 - \frac{v}{r_d^3} \frac{r_d}{r} e^{-r/r_d}, \quad (1)$$

which describes its behavior well at sufficiently large distances. However, at small distances expression (1) becomes negative because the correction term predominates over the principal term, which indicates the necessity of improving the convergence of the expansion at small distances.

A number of attempts ⁽²⁾ were undertaken in this direction, the essence of which was basically to replace, for one reason or another, the Coulomb interaction by some other interaction not leading to such difficulties. It seems to us that attempts of this kind, despite a certain effectiveness in carrying out concrete calculations, do not answer the question of the causes of the difficulties in constructing the radial function for systems with purely Coulomb interaction. Below we set ourselves the goal of improving the convergence of N. N. Bogolyubov' s expansion at small distances.

For $G(r)$, N. N. Bogolyubov obtained, approximating the ternary distribution function by a product of three binary ones, the following system of nonlinear integral equations ⁽¹⁾:

$$W(|q|) = \Phi(|q|) + \frac{1}{v} \int dq_1 \{G(|q_1|) - 1\} \int_{\infty}^{|q-q_1|} dr G(r) \frac{d\Phi(r)}{dr};$$

$$G(|q|) = \exp \left\{ -\frac{1}{\theta} W(|q|) \right\} \quad (2)$$

with respect to the functions G and W , where $\Phi(|q|) = \frac{e^2}{|q|}$.

Let us make the following substitution of the unknown functions:

$$G(|q|) = e^{-\bar{\Phi}(|q|)/\theta} C(|q|); \quad W(|q|) = \bar{\Phi}(|q|) + V(|q|), \quad (3)$$

where $\bar{\Phi}(r)$ is some function, which for the time being we require to satisfy when as $r \rightarrow 0$, $\bar{\Phi}(r) \sim e^2/r$. In this case system (2) takes the form

$$\begin{aligned} V(|q|) &= \Phi(|q|) - \bar{\Phi}(|q|) + \\ &+ \frac{1}{v} \int dq_1 \left\{ e^{-\bar{\Phi}(|q_1|)/\theta} C(|q_1|) - 1 \right\} \int_{\infty}^{|q-q_1|} dr \frac{d\bar{\Phi}(r)}{dr} e^{-\bar{\Phi}(r)/\theta} C(r); \quad (4) \\ C(|q|) &= \exp \left\{ -\frac{1}{\theta} V(|q|) \right\}. \end{aligned}$$

To solve this system we apply the “plasma” expansion ⁽¹⁾, i.e., we set

$$\frac{1}{\theta} \Phi(|q|) = v\psi(|q|); \quad \frac{1}{\theta} \bar{\Phi}(|q|) = v\bar{\psi}(|q|), \quad (5)$$

and shall seek V and C in the form of series in powers of v :

$$\begin{aligned} C(|q|) &= C_0(|q|) + vC_1(|q|) + v^2C_2(|q|) + \dots; \\ V(|q|) &= V_0(|q|) + vV_1(|q|) + v^2V_2(|q|) + \dots. \end{aligned} \quad (6)$$

The equations of the zeroth approximation have the form

$$\begin{aligned} V_0(|q|) &= \theta \int dq_1 \{ C_0(|q_1|) - 1 \} \int_{\infty}^{|q-q_1|} dr \frac{d\bar{\psi}(r)}{dr} C_0(r); \\ C_0(|q|) &= \exp \left\{ -\frac{1}{\theta} V_0(|q|) \right\}. \end{aligned} \quad (7)$$

They evidently have the solution

$$V_0(|q|) = 0; \quad C_0(|q|) = 1. \quad (8)$$

Taking (8) into account, the equations of the first approximation have the form

$$\begin{aligned}
 V_1(|q|) &= \theta\{\psi(|q|) - \bar{\psi}(|q|)\} + \\
 &+ \theta \int dq_1 \{C_1(|q_1|) - \bar{\psi}(|q_1|)\} \psi(|q - q_1|); \\
 C_1(|q|) &= -\frac{1}{\theta} V_1(|q|).
 \end{aligned} \tag{9}$$

Hence, for the first-order correction we obtain

$$C_1(|q|) = \bar{\psi}(|q|) - \frac{1}{4\pi r_d^2} \frac{e^{-|q|/r_d}}{|q|}. \tag{10}$$

We now define $\bar{\psi}$ additionally from the condition that the first-order correction C_1 vanish; then

$$\bar{\Phi}(|q|) = \frac{e^2}{|q|} e^{-|q|/r_d}, \tag{11}$$

i.e., $\bar{\Phi}(|q|)$ turns out to be the Coulomb potential with Debye screening.

The solution of the equations of the second approximation has the form

$$\begin{aligned}
 C_2(|q|) &= -\frac{1}{2} \int dq_1 \bar{\psi}(|q - q_1|) \bar{\psi}^2(|q_1|) \\
 &- \int dq_1 \bar{\psi}(|q - q_1|) \int_{\infty}^{|q_1|} dr \frac{d\psi(r)}{dr} \bar{\psi}(r) + \int dq_2 \bar{\psi}(|q - q_2|) \int dq_1 \bar{\psi}(|q_2 - q_1|) \int_{\infty}^{|q_1|} dr \frac{d\psi(r)}{dr} \bar{\psi}(r).
 \end{aligned} \tag{12}$$

It is not difficult to see that the correction C_2 is finite for all q , since, although the first two integrals in this formula diverge for small q , they nevertheless compensate each other, while the last integral is finite for small q .

Neglecting corrections of the second approximation and higher, we obtain for the radial function the expression

$$G(r) = \exp \left\{ -\frac{e^2}{\theta} \frac{1}{r} e^{-r/d} \right\}. \tag{13}$$

The expression obtained for G goes over at large distances into formula (1), and at small distances into the Boltzmann formula

$$G(r) \sim \exp \left\{ -\frac{e^2}{\theta r} \right\}.$$

Let us show that (13) can be obtained without using the ternary approximation, directly from the chain of equations for the distribution functions F_s (1):

$$\frac{\partial F_s}{\partial q_1^\alpha} + \frac{1}{\theta} \frac{\partial U_s}{\partial q_1^\alpha} F_s + \frac{1}{\theta v} \int \frac{\partial \Phi(|q_1 - q_{s+1}|)}{\partial q_1^\alpha} F_{s+1} dq_{s+1} = 0 \quad (s = 1, 2, \dots) \quad (14)$$

under the known condition of weakening of correlations and the normalization condition for the functions F_s . We make the substitution of the unknown functions F_s :

$$F_s = e^{-\bar{U}_s/\theta} C_s, \quad (15)$$

where

$$\bar{U}_s = \sum_{(1 \leq i < j \leq s)} \bar{\Phi}(|q_i - q_j|);$$

$\bar{\Phi}(r)$ for $r \rightarrow 0$ is of order e^2/r . For C_s we obtain the chain of equations

$$\begin{aligned} & \frac{\partial C_s}{\partial q_1^\alpha} + \frac{1}{\theta} \left(\frac{\partial U_s}{\partial q_1^\alpha} - \frac{\partial \bar{U}_s}{\partial q_1^\alpha} \right) C_s + \\ & + \frac{1}{\theta v} \int \frac{\partial \Phi(|q_1 - q_{s+1}|)}{\partial q_1^\alpha} \exp \left[-\frac{1}{\theta} \sum_{(1 \leq i \leq s)} \bar{\Phi}(|q_i - q_{s+1}|) \right] C_{s+1} dq_{s+1} = 0 \end{aligned} \quad (16)$$

with an analogous condition of weakening of correlations and the normalization condition.

Following (1), we apply to the solution of (16) the “plasma” expansion, in accordance with which we make the substitution

$$\frac{1}{\theta} \Phi(r) = v\psi(r); \quad \frac{1}{\theta} \bar{\Phi}(r) = v\bar{\psi}(r), \quad (17)$$

and the functions C_s will be sought in the form

$$\begin{aligned} C_1(q) &= g_1(q); \\ C_2(q_1, q_2) &= g_1(q_1)g_1(q_2) + \vartheta g_2(q_1, q_2); \\ C_3(q_1, q_2, q_3) &= g_1(q_1)g_1(q_2)g_1(q_3) + \\ &+ \vartheta \{g_2(q_1, q_2)g_1(q_3) + g_2(q_2, q_3)g_1(q_1) + g_2(q_3, q_1)g_1(q_2)\} + \\ &+ \vartheta^2 g_3(q_1, q_2, q_3), \end{aligned} \quad (18)$$

where g_s should be sought in the form of series in powers of ϑ .

It may be noted that, owing to the spatial homogeneity of the problem, $g_1(q) = 1$, $g_2(q_1, q_2) = g_2(|q_1 - q_2|)$; in this case the first of equations (16) is satisfied identically. For $g_2^0(|q|)$ we obtain the equation

$$\frac{\partial g_2^0(|q|)}{\partial q^\alpha} + \frac{\partial \psi(|q|)}{\partial q^\alpha} - \frac{\partial \bar{\psi}(|q|)}{\partial q^\alpha} + \int dq' \frac{\partial \psi(|q - q'|)}{\partial q^\alpha} (g_2^0(|q'|) - \bar{\Psi}'(|q'|)), \quad (19)$$

whose solution will be

$$g_2^0(|q|) = \bar{\psi}(|q|) - \frac{1}{4\pi r_d^2} \frac{1}{|q|} e^{-|q|/r_d}, \quad (20)$$

which coincides identically with (10) and, consequently, leads for the radial function to expression (13) when corrections of order ϑ^2 and higher are neglected.

One may hope that the considerations given above, with some modifications, can be applied to the problem of a system of charged particles of different signs.

In conclusion, we take the opportunity to express our gratitude to Academician N. N. Bogolyubov for a valuable discussion of the work.

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References Cited

- ¹ N. N. Bogolyubov, *Problems of Dynamical Theory in Statistical Physics*, Moscow, 1946.
- ² A. E. Glauberman, DAN, **78**, 883 (1951); A. E. Glauberman, I. R. Yukhnovskii, ZhETF, **22**, 562 (1952); ZhETF, **22**, 572 (1952); I. R. Yukhnovskii, ZhETF, **27**, 690 (1954).

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