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Abstract

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EXACT SOLUTION OF A NONLINEAR PROBLEM ON AN EXPLOSION IN A GAS WITH VARIABLE INITIAL DENSITY

(Presented by Academician L. I. Sedov on 22 VI 1957)

Let, at the initial instant of time $t = 0$, in a gas at rest, at a point, along a straight line, or along a plane, a finite energy E be instantaneously released, i.e., an explosion occur. In the case of a cylindrical charge the energy E is reckoned per unit length, and in the case of a plane charge, per unit area ([1]). A spherical, cylindrical, or plane explosive shock wave propagates through the gas. Behind the shock wave there occurs an unsteady one-dimensional motion of the gas with spherical, cylindrical, or plane symmetry.

The initial pressure p_1 is constant; the initial density of the gas is variable and changes with the distance from the center of the explosion according to the law

$$\rho_1(r) = \frac{a(\gamma-1)^2}{\gamma\left(\frac{\gamma+1}{2}\right)^{\beta-1}\left(\frac{r}{r^0}\right)^{\omega}\left[\left(\frac{r}{r^0}\right)^{\nu}+\frac{\nu(\gamma^2-1)}{2\sigma_\nu\gamma}\right]^{\beta}}, \tag{1}$$

where γ is the ratio of specific heats; a is an arbitrary positive constant;

$$\omega = \frac{\nu(3-\gamma)+2\gamma-2}{\gamma+1}; \qquad \beta = \frac{3\nu\gamma+4-\nu}{\nu(\gamma+1)}; \qquad r^0 = \left(\frac{E}{p_1}\right)^{1/\nu}$$

is the dynamic length; $\nu = 3, 2, 1$, respectively for the case of a spherical, cylindrical, or plane wave; $\sigma_3 = 4\pi$; $\sigma_2 = 2\pi$; $\sigma_1 = 2$. From (1) it is seen that ρ_1 depends parametrically on the value of γ and on the dynamic length r^0.

One-dimensional adiabatic motions of the gas behind the wave are described by the system of equations

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0;$$

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial v}{\partial r} + \frac{v(\nu-1)}{r}\right) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right)+v\frac{\partial}{\partial r}\left(\frac{p}{\rho^{\gamma}}\right)=0,$$

where v is velocity, p is pressure, and ρ is density. It is required to determine the dependence of the velocity, pressure, and density of the gas on the linear coordinate r and time t, as well as the dependence of the radius of the shock wave r_2 on time.

The problem reduces to finding the solution of system (2) with the initial conditions indicated above, and also with the boundary condition at the center of symmetry $v(0,t)=0$ and the conditions at the front of the explosive wave, which may be written in the form ([1])

$$v_2=\frac{2c}{\gamma+1}(1-q),\qquad p_2=\frac{p_1}{\gamma+1}\,\frac{2\gamma-(\gamma-1)q}{q},$$

$$\rho_2=\frac{\rho_1(\gamma+1)}{\gamma-1+2q},\tag{3}$$

where $c=dr_2/dt$ is the velocity of the shock wave; $q=\gamma p_1/\rho_1 c^2$. By direct verification one can make sure that the solution of the posed problem is given by the formulas

$$v=\frac{r}{kt},\qquad k=\frac{\nu(\gamma-1)+2}{2};\tag{4}$$

$$p=\frac{p_1|kt|^{-\gamma\nu/k}}{\gamma+1}\left\{\frac{4\gamma}{b(\gamma+1)}[f(x)]^{-(\gamma-1)/2}-(\gamma-1)[f(x)]^{-\gamma}\right\};\tag{5}$$

$$\rho=\frac{2p_1|kt|^{-(\nu-1)/k}}{r\nu(\gamma^2-1)}\,\frac{d}{dx}\left\{\frac{4\gamma}{b(\gamma+1)}[f(x)]^{-(\gamma-1)/2}-(\gamma-1)[f(x)]^{-\gamma}\right\};\tag{6}$$

$$\left(\frac{r_2}{r_0}\right)^{\nu}=\frac{\gamma^2-1}{2\gamma\sigma_\nu\left\{\frac{2}{(\gamma+1)b}|kt|^{-\nu(\gamma+1)/2k}-1\right\}},\tag{7}$$

where $x=r|kt|^{-1/k}$; $b=\left[\frac{\nu^2 p_1}{(\sigma_\nu r^0)^2 a}\right]^{-1/(\beta-1)}$; $f(x)\geq 0$ is a function taking no negative values. The dependence $f(x)$ is determined from the equation

$$\left(\frac{x}{r^0}\right)^{\nu}+\frac{\gamma^2-1}{2\sigma_\nu\gamma}f-\frac{2}{(\gamma+1)b}\left(\frac{x}{r^0}\right)^{\nu}f^{(\gamma+1)/2}=0.\tag{8}$$

The change in pressure immediately behind the front of the shock wave is given by the formula

$$p_2 = p_1 \left[1 + \frac{\nu(\gamma - 1)}{\sigma_\nu} \left(\frac{r^0}{r^2} \right)^\nu \right].$$

The indicated solution was obtained from the exact solution of L. I. Sedov ([2]). The method of constructing discontinuous solutions for this exact solution was developed by the author of the present note jointly with E. V. Ryazanov.

From the solution found by us, in the special case $p_1 = 0, b = 0$, we obtain the known solution ([1]) of the self-similar problem of a point explosion, when the initial density is distributed according to the law $\rho_1 = Ar^{-\omega}$, where A is a certain constant.

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CITED LITERATURE

1. L. I. Sedov, *Methods of Similarity and Dimension in Mechanics*, Moscow, 1957.

2. L. I. Sedov, DAN, **90**, No. 5 (1953).

Note: Figure translations are in progress. See original paper for figures.