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Abstract

Full Text

Physics

Yu. N. Dnestrovsky and D. P. Kostomarov

Radiation of Charged Particles Passing Near Ideally Conducting Bodies

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In connection with the development of ultrahigh-frequency technology and questions of the generation of millimeter waves, interest has increased in recent years in problems concerning the radiation of charged particles passing near dielectric or conducting bodies⁽¹⁻⁴⁾. The particular problem of the radiation of a point charge moving uniformly near an ideally conducting sphere was considered in work⁽⁵⁾. The general problem of the radiation of charged particles passing near ideally conducting bodies is investigated below in the nonrelativistic approximation.

Let us consider the problem of the radiation of a point charged particle of mass m and charge e as it passes near an ideally conducting surface S . We shall assume that the surface S possesses axial symmetry, so that its equations have the form $r = h_1(s)$, $z = h_2(s)$, where s is the arc length ($-\infty < s < \infty$), with $r(s) \neq 0$, $\lim_{s \rightarrow \pm\infty} r(s) \neq 0$. The charge moves along the axis of the system from negative to positive values of z .

In an exact formulation the problem is very complicated. In the present note we shall restrict ourselves to the study of the nonrelativistic approximation, when the stated problem can be divided into two:

I. Neglecting retardation, integrate the equation of motion of the charge and find the magnitude of the charges and currents induced on the screen (the electric field is then considered as electrostatic, depending on time as on a parameter).

II. Calculate the radiation of the system of currents determined by the solution of problem I.

Mathematically, problem I reduces to the following boundary-value problem:

$$\Delta u = 0 \quad \text{in } T, \quad u|_S = -u_0|_S, \quad (1)$$

$$m\ddot{z}_0 = -e \frac{\partial u}{\partial z}(0, z, z_0) \Big|_{z=z_0}, \quad \lim_{t \rightarrow -\infty} \dot{z}(t) = v_0, \quad (2)$$

where $u_0(r, z, z_0)$ is the Coulomb potential of the charge e placed at the point $M_0(0, z_0)$. The function $u(r, z, z_0)$, up to the factor $4\pi e$, is the regular part g of the Green's function G of the Dirichlet problem for the region T bounded by the surface S .

For what follows it is convenient to introduce dimensionless coordinates:

$$\rho = \frac{r}{a}, \quad \zeta = \frac{z}{a}, \quad \zeta_0 = \frac{z_0}{a}, \quad \alpha = \frac{s}{a},$$

where a is some characteristic dimension of the system, for example, the minimum radius of the channel or aperture through which the charge e passes. The equations of the surface S in the new coordinates take the form

$$\rho = \rho(\alpha) = \frac{1}{a}h_1(a\alpha), \quad \zeta = \zeta(\alpha) = \frac{1}{a}h_2(a\alpha).$$

Integrating the equation of motion (2), we obtain an expression for the velocity of the charge

$$v(\zeta_0) = \sqrt{v_0^2 + \frac{2e^2}{am}f(\zeta_0)}; \quad (3)$$

here

$$f(\zeta_0) = \int_{-\infty}^{\zeta_0} \varphi(\varepsilon) d\varepsilon, \quad \varphi(\zeta_0) = -4\pi \left. \frac{\partial g}{\partial \zeta}(0, \zeta, \zeta_0) \right|_{\zeta=\zeta_0}.$$

The densities of the charges $\sigma(\alpha, \zeta_0)$ and currents $j(\alpha, \zeta_0)$ induced on the screen S are determined by the relations

$$\sigma(\alpha, \zeta_0) = \frac{e}{a^2}\sigma_0(\alpha, \zeta_0) = \frac{e}{a^2} \frac{\partial G}{\partial \nu}(\rho(\alpha), \zeta(\alpha), \zeta_0)$$

$$(\nu = n/a, \quad n\text{—the outward normal}),$$

$$j(\alpha, \zeta_0) = \frac{ev(\zeta_0)}{a^2}j_0(\alpha, \zeta_0) = \frac{ev(\zeta_0)}{a^2} \frac{1}{\rho(\alpha)} \int_{\alpha}^{\infty} \frac{\partial \sigma_0(\varepsilon, \zeta_0)}{\partial \zeta_0} \rho(\varepsilon) d\varepsilon.$$

Since the law of motion of the charge and the magnitude of the currents induced by it on the screen have been found, it remains to calculate the radiation of the specified system of currents. Restricting ourselves to the dipole approximation, we represent the total dipole moment of the system $p(t)$ as the sum of the

moments of the charge $p_1(t)$ and of the screen $p_2(t)$. For the second derivatives \ddot{p}_1 and \ddot{p}_2 we obtain the expressions

$$\ddot{p}_1 = e\ddot{z}_0, \quad \ddot{p}_2 = \frac{dJ}{dt};$$

here J is the total current of the screen

$$J = ev(\zeta_0)\chi(\zeta_0), \quad \text{where} \quad \chi(\zeta_0) = 2\pi \int_{-\infty}^{\infty} j_0(\alpha, \zeta_0)\rho(\alpha) \frac{d\zeta(\alpha)}{d\alpha} d\alpha,$$

and, consequently,

$$\frac{dJ}{dt} = \frac{e^3}{a^2m}\varphi(\zeta_0)\chi(\zeta_0) + \frac{ev^2(\zeta_0)}{a}\psi(\zeta_0), \quad \text{where} \quad \psi(\zeta_0) = \chi'(\zeta_0).$$

Thus,

$$\ddot{p} = \frac{e^3}{a^2m}(1 + \chi(\zeta_0))\varphi(\zeta_0) + \frac{ev^2(\zeta_0)}{a}\psi(\zeta_0),$$

and we arrive at the following formulas for the power w and the total radiation E :

$$w = \frac{2}{3c^3}|\ddot{p}|^2 = \frac{2}{3} \frac{e^2c}{a^2} \{ \gamma^2(1 + \chi)^2\varphi^2 + 2\gamma(1 + \chi)\varphi\psi[\beta^2 + 2\gamma f] + \psi^2[\beta^2 + 2\gamma f]^2 \}, \quad (4)$$

$$E = \int_{-\infty}^{\infty} w dt = \frac{2}{3} \frac{e^2}{a} \left\{ \gamma^2 \int_{-\infty}^{\infty} \frac{(1 + \chi)^2\varphi^2 d\zeta_0}{\sqrt{\beta^2 + 2\gamma f}} + 2\gamma \int_{-\infty}^{\infty} (1 + \chi)\varphi\psi\sqrt{\beta^2 + 2\gamma f} d\zeta_0 + \int_{-\infty}^{\infty} \psi^2[\beta^2 + 2\gamma f]^{3/2} d\zeta_0 \right\} \quad (5)$$

here $\beta = v_0/c$, $\gamma = e^2/amc^2$. For one electron $\gamma = r_0/a$, where $r_0 = e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm is the classical radius of the electron. In the case of the passage of a "clump" of N particles $\gamma = \gamma_N = Nr_0/a$.

Both dimensionless parameters β and γ in formulas (4) and (5) are small; however, by changing v_0 , we shall change the ratio between them.

Let us consider two limiting cases:

1. $\beta^2 \ll \gamma$. Then from formula (3) we find $v(\xi_0) \simeq \sqrt{2}c\sqrt{\gamma f(\xi_0)}$, and as a result the expression for the total radiation E takes the form

$$E = \frac{4\sqrt{2}}{3} \frac{e^2}{a} \gamma^{3/2} \left\{ \frac{1}{4} \int_{-\infty}^{\infty} \frac{(1+\chi)^2 \varphi^2 d\xi_0}{\sqrt{f}} + \int_{-\infty}^{\infty} (1+\chi) \varphi \psi \sqrt{f} d\xi_0 + \int_{-\infty}^{\infty} \psi^2 f^{1/2} d\xi_0 \right\}. \quad (6)$$

All three terms in expression (6) have the same order; consequently, allowance for the nonuniformity of the motion of the charge e under the condition $\beta^2 \ll \gamma$ proves essential for calculating the total radiation. In this case the radiation spectrum consists of waves whose length satisfies the condition $\lambda \gg \lambda_1$, where $\lambda_1 \sim a/\sqrt{\gamma} \gg a$.

2. $\gamma \ll \beta^2$. Then, according to formula (3), $v(\xi_0) \simeq v_0 = \text{const}$, i.e. the motion of the charge may be regarded as uniform. Formula (5), taking into account the relations

$$\frac{E_1}{E_3} \sim \left(\frac{\gamma}{\beta^2} \right)^2 \ll 1, \quad \frac{E_2}{E_3} \sim \frac{\gamma}{\beta^2} \ll 1,$$

takes the form

$$E \simeq E_3 = \frac{2}{3} \frac{e^2}{a} \beta^3 \int_{-\infty}^{\infty} \psi^2(\xi_0) d\xi_0. \quad (7)$$

To determine the spectrum, we expand the function $\ddot{p}(t) \simeq \ddot{p}_2(t)$ in a Fourier integral

$$P(\mu) \simeq P_2(\mu) = e v_0 \Psi(\mu), \quad \text{where} \quad \Psi(\mu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\mu\xi_0} \psi(\xi_0) d\xi_0 \left(\mu = \frac{\omega a}{v_0} \right). \quad (8)$$

Thus, in this case the waves emitted are mainly those whose length satisfies the condition $\lambda \gg \lambda_2$, where $\lambda_2 \sim a/\beta \gg a$.

The analysis carried out permits the following conclusions:

1. The total radiation increases with increasing initial velocity as v_0^3 .
2. The spectrum consists mainly of waves whose length considerably exceeds a ; however, as v_0 increases, the "boundary" of the emitted spectrum shifts toward shorter waves.
3. The lower limit of applicability of the approximation of prescribed currents (the assumption of constancy of the charge velocity) is determined by the inequality

$$\frac{\gamma}{\beta^2} = 2 \frac{e^2/a}{T_0} \ll 1, \quad \text{where} \quad T_0 = \frac{m v_0^2}{2}.$$

Fig. 1

Figure 1: Fig. 1

The question of the upper limit can be resolved only on the basis of an analysis of the relativistic problem.

Let us now consider the case in which the system is excited not by a separate point charge, but by a modulated beam of electrons moving with constant velocity v_0 , and assume that their linear density on the z -axis has the form

$$\eta(z, t) = \frac{I_0}{v_0} e^{i(\omega t - k_0 z)} \quad \left(k_0 = \frac{\omega}{v_0} \right).$$

The assumption of constancy of the velocity means that we neglect the interaction of the electrons with the screen and with one another. As calculations show, this assumption is valid if

$$\delta = 2 \frac{I_0 e}{m v_0^3} \ll 1.$$

This inequality proves to be satisfied over a rather wide range of velocities and currents used. Thus, for $I_0 = 0.1$ A, $\beta = 0.1$ we find $\delta = 0.01$.

The radiation of the system in the present case will be monochromatic, with a frequency equal to the excitation frequency ω . To calculate the radiation power, one should find the second time derivative of the dipole moment, which has the form

$$\ddot{p} \simeq \ddot{p}_2 = I_0 v_0 \Psi(\mu) e^{i\omega t},$$

where $\Psi(\mu)$ is the Fourier transform of the function $\psi(\zeta_0)$ (8). For the radiation resistance R , etc., of the system x we obtain the expressions

$$R = 80\pi\beta^2 |\Psi(\mu)|^2 \text{ ohm}; \quad (9)$$

$$x = \frac{I_0^2 R}{I_0 V} = 0.98 \cdot 10^{-3} |\Psi(\mu)|^2 I_0; \quad (10)$$

here V is the accelerating potential in volts; I_0 is the current in amperes.

Fig. 1

We have carried out calculations of the radiation upon the entry of a beam of particles from an open half-space into a circular waveguide of radius a with an

Fig. 2

Figure 2: Fig. 2

infinite flange (Fig. 1). The function $\Psi(\mu)$ is then approximately represented by the expression

$$\Psi(\mu) \simeq 0.54e^{-0.83|\mu| - i0.09\mu} \sin(0.35|\mu| + 0.4).$$

Figs. 2A and 2B show a very sharp dependence of the radiation resistance on the initial velocity, frequency, and channel radius.

Fig. 2. Dependence of the radiation resistance: A —on β ; B —on a/λ

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Moscow State University
named after M. V. Lomonosov

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