PHYSICS

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1957

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Abstract

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DISPERSION RELATIONS IN CASES OF WEAK INTERACTION

At present, much attention is being devoted to dispersion relations in connection with the possibility of experimentally establishing in this way the fact of the existence of an elementary length. In this connection it is of interest to analyze what these relations give in cases of weak interaction. Let us consider, as an example, the reaction

$$\mu + p \to n + \nu, \tag{1}$$

in which, alongside the weak interaction, there is a strong interaction of the nucleon and meson fields.

In the theory of dispersion relations the amplitude of a process is split into Hermitian and anti-Hermitian parts in a certain sense. In a coordinate system in which the sum of the nucleon momenta before and after the reaction is equal to zero, the "Hermitian part" D is equal to a certain integral of the "anti-Hermitian part" A plus an arbitrary polynomial $P_n(E)$ in the energy E of the incident particle. The anti-Hermitian part of the amplitude is expressed through the product of the "meson" and "neutrino" currents. Because of the smallness of the coupling constant of the weak interaction C, one may take into account terms containing the coupling constant only to first order. The product of the indicated currents, however, is of smallness no lower than second order, and therefore in the present approximation the anti-Hermitian part is equal to zero*. Consequently, the dispersion relations acquire an especially simple form:

$$D(E) = P_n(E). \tag{2}$$

As is known, the principal difficulties in proving dispersion relations arise from the need to analyze the analytic properties of A. Since in the case under consideration A is identically equal to zero, the proof becomes completely obvious and may be easily obtained both from general principles and from the ordinary theory.

Let us write the matrix element of process (1) in the form

$$S(p, q; p', q') = (2\pi)^3 \left\langle \Phi^*_{p's'} a_\nu^{(-)}(q') S a_\mu^{(+)}(q) \Phi_{ps} \right\rangle, \tag{3}$$

where Φ_{ps} is the state vector of the initial nucleon; $a_\mu^{(+)}$ is the creation operator of the μ^--meson; $a_\nu^{(-)}$ is the absorption operator of the neutrino.

Moving in (3) the creation operator to the extreme left, and the annihilation operator to the extreme right, we obtain

$$S(p,q;p',q') = \bar{u}_\nu(q') \int e^{i(q'x-qy)} \left\langle p's' \left| \frac{\delta^2 S}{\delta\bar{\psi}_\nu(x)\,\delta\psi_\mu(y)} S^+ \right| ps \right\rangle dx\,dy\,u_\mu(q),$$

where ψ_μ is the μ-meson field; ψ_ν is the neutrino field.

(4)

* The case of electromagnetic interactions requires special consideration, since in this case the matrix elements in first approximation may be proportional both to e and to e^2.

We introduce the operators of the "meson" and "neutrino" currents:

$$j_\mu(y) = -i\,\frac{\delta S}{\delta\psi_\mu(y)}\,S^+; \qquad j_\nu(x) = -i\,\frac{\delta S}{\delta\bar{\psi}_\nu(x)}\,S^+. \tag{5}$$

Then we have

$$\frac{\delta^2 S}{\delta\bar{\psi}_\nu(x)\delta\psi_\mu(y)}\,S^+ = \begin{cases} -i\,\dfrac{\delta\bar{j}_\mu(y)}{\delta\bar{\psi}_\nu(x)} - \bar{j}_\mu(y)j_\nu(x), \\ -i\,\dfrac{\delta j_\nu(x)}{\delta\psi_\mu(y)} + j_\nu(x)\bar{j}_\mu(y). \end{cases} \tag{6}$$

But, by virtue of the causality principle (2),

$$\frac{\delta j_\nu(x)}{\delta\psi_\mu(y)} = 0, \quad y \lesseqgtr x; \qquad -\frac{\delta\bar{j}_\mu(y)}{\delta\bar{\psi}_\nu(x)} = 0, \quad x \lesseqgtr y. \tag{7}$$

Therefore, taking the first term in the expansion in the weak-interaction constant C, we obtain:

$$\left\{ \frac{\delta^2 S}{\delta\bar{\psi}_\nu(x)\delta\psi_\mu(y)}\,S^+ \right\}_C = 0 \quad \text{for } x \neq y. \tag{8}$$

This means that the given expression is a quasilocal operator and contains $\delta(x-y)$ and, possibly, its derivatives. If one makes the usual assumption that derivatives of the meson and neutrino fields do not enter the interaction Lagrangian, then

$$\left\{ \frac{\delta^2 S}{\delta \bar{\psi}_\nu(x) \delta \psi_\mu(y)} S^+ \right\}_C = \delta(x-y)\Lambda(x). \tag{9}$$

Taking (8) into account, we find

$$\int e^{i(q'x-qy)} \left\langle p's' \left| \frac{\delta^2 S}{\delta \bar{\psi}_\nu(x) \delta \psi_\mu(y)} S^+ \right| ps \right\rangle dx\, dy =$$

$$= \int e^{i(q'-q)x} \langle p's'|\Lambda(x)|ps\rangle dx =$$

$$= (2\pi)^4 \delta(p'+q'-p-q) \langle p's'|\Lambda(0)|ps\rangle. \tag{10}$$

Expression (10) depends on the momenta p and $q'-q$ and does not depend on the momentum $q+q'$. If one uses a reference frame in which $p+p'=0$, this means that expression (10) is a function of p and does not depend on E. Thus, the unknown functions of the process amplitude, determined by the strong interactions, depend only on the momentum transfer to the nucleon.

Substituting (10) into (4) and taking into account considerations of relativistic invariance, we obtain

$$S(p,q;p',q') = \sum_{\alpha,\beta} (\bar{u}_\nu(q') O^\alpha u_\mu(q))(\bar{u}_n(p') \Omega_\alpha^\beta u_p(p)) F_{\alpha\beta}(|p-p'|). \tag{11}$$

The fact established above that the functions $F_{\alpha\beta}$ are independent of the momentum $q+q'$ can also be obtained from the ordinary theory, if one uses graph techniques. The most general form of diagrams contributing to process (1) is shown in Fig. 1.

Since the meson and neutrino lines converge at a single point in the diagrams (which in the present case reflects the locality condition), we see that the part of the graph containing the strong interactions depends only on p and $p-p'$, and does not depend on the momentum $q+q'$, i.e., we again arrive at expression (11). It is not difficult to note that if the causality condition is violated and a form factor is inserted between the corresponding lines in Fig. 1, then a substantial dependence of the amplitude under study on $q+q'$ will also appear.

Considering expression (11), we see that, for a fixed momentum transfer to the nucleon, the energy dependence of the differential cross section of reaction (1) is determined by known matrix elements and by the constants $F_{\alpha\beta}$. Experimental determination of this dependence may serve as a direct test of the causality principle in process (1).

Fig. 1

The study of the dependence of the functions $F_{\alpha\beta}$ on the momentum transfer to the nucleon could make it possible to determine the "meson-neutrino" structure of the nucleon.

Comparing the diagrams in Fig. 1 with the diagrams for the process of electron scattering on a nucleon, one may conclude that the effective size of the "meson-neutrino" structure of the nucleon is apparently of the same order as that of the "electromagnetic" structure.

It should be emphasized that the results obtained above are fully applicable to β-decay and to such decays of hyperons and K-mesons in which, along with strongly interacting particles, μ-, e-, and ν-particles participate.*

Of particular interest from the point of view of testing the causality principle is the study of the angular and energy distributions of electrons and μ-mesons in decays of hyperons and K-mesons, since in these processes μ- and e-particles can have comparatively large energies.

Joint Institute for Nuclear Research

Received
22 V 1957

REFERENCES

1. N. N. Bogolyubov, B. V. Medvedev, M. K. Polivanov, *Dispersion Relations*, Moscow, 1957.

2. N. N. Bogolyubov, *Izv. AN SSSR, Ser. Fiz.*, **19**, 237 (1955).

* Let us note, incidentally, the special role of the reaction $\mu^- \to e^- + \nu + \tilde{\nu}$. Since only weakly interacting particles participate in it, the locality condition is equivalent to the applicability of the first approximation of the usual perturbation theory.

Note: Figure translations are in progress. See original paper for figures.