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Abstract

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SOME QUESTIONS IN THE DYNAMICS OF FLIGHT TO THE MOON

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With the present development of rocketry, the attainment of velocities sufficient not only for creating an artificial satellite of the Earth, but also for flight to the Moon, is becoming realistic. But up to now the literature [(1–5)] has not provided a satisfactory solution of the fundamental questions of the theory of flight to the Moon: the form and classification of trajectories on the passive segment, the possibility of flying around the Moon with return to Earth, the possibility of periodic flight around the Moon and the Earth, the question of hitting the Moon, and also the especially important question of the influence of the scatter of initial data on the realization of a hit or flyby. This is explained by fundamental difficulties. Indeed, in the simplest formulation, taking into account only the principal forces acting on the rocket, the problem reduces to the unsolved circular restricted three-body problem in mechanics (m_1—Earth, m_2—Moon, m_0—rocket). In 1953-1955 we undertook an attempt at a systematic investigation of the planar problem. To find the trajectories of interest to us and to determine the influence of the scatter of initial data, theoretical methods were supplemented by numerical ones, with the use of electronic machines. Below we set forth the main results of the solution of the problems indicated above.*

1. **The problem of hitting the Moon.** A method has been found for determining initial data corresponding to hits. It has been rigorously proved that the minimum distance ρ of the rocket from the center of the Moon is a quadratic, and not a linear, function of the errors in the initial data, so that an exact hit on the Moon is an easier problem than hitting an attractive point moving in the same way as the Moon. For the value of the angle α_1 between the initial geocentric radius r_1 and the velocity V_1, equal to $\pi/2$, dependences on V_1 have been found: for the flight time, the angular distance between the starting point and the point of impact, and for the coefficients k_i in the expressions $\rho_i = k_i(\delta x_i)^2$ (where δx_i is the variation of the i-th initial datum) in the range $-0.1 < \Delta V_1 < 0.5$ km/sec, where $\Delta V_1 = V_1 - V_\text{p}$, and $V_\text{p}^2 = 2fm_1/r_1$. It turned out that the minimum ΔV_1 corresponding to impact belong to this range and that the course of the dependences for $\Delta V_1 > 0.5$ is already asymptotic (to the values corresponding to $\Delta V_1 = \infty$). It also turned out that neglect of the Moon's attraction gives $\rho \leq 1$ km if $\Delta V_1 \geq 0$. As ΔV_1 decreases, the miss distance ρ grows rapidly. The least favorable, in terms of accuracy, are velocities

Fig. 1

Figure 1: Fig. 1

close to the minimum; the most favorable are velocities somewhat smaller than V_p. For $V_1 < V_p$, the Moon can be hit both on the ascending and on the descending branch of the trajectory, but the accuracy of impact on the descending branch is 2–5 times worse than the accuracy of impacts on the ascending branch for the same ΔV_1. The influence of an error in the initial altitude proved to be practically negligible in all cases.

Example. Errors $\delta V_1 = 10$ m/sec, $\delta\alpha_1 = 10^{-2}$, $\delta r_1 = 50$ km cause,

* Obtained by the author at the Mathematical Institute of the Academy of Sciences of the USSR and reported there in February 1956.

respectively, the following misses (in km) on the ascending branch: 1) for $\Delta V_1 = -0.092828$: $\rho_v = 11184,\ \rho_\alpha = 1208,\ \rho_r = 56$; 2) for $\Delta V_1 = 0$: $\rho_v = 125,\ \rho_\alpha = 5159,\ \rho_r = 43$; 3) for $\Delta V_1 = +0.106093$: $\rho_v = 318,\ \rho_\alpha = 6121,\ \rho_r = 140$.

2. **The problem of flying around the Moon.** A method has been found for determining initial data corresponding to a flyby of the Moon, with return to the point m_1. Here, too, the miss on return is a quadratic function of the errors in the initial data. A flyby of the Moon in the plane ξ, η, rotating together with the straight line $m_1 m_2$ (see Fig. 1), is possible only clockwise, i.e., in the direction opposite to the velocity of its revolution.

Fig. 1

Flyby trajectories, according to the character of the approach to m_2, may be of two classes:

1) A close approach occurs for all α_1 on the ascending ( ) branch, which immediately passes into the descending ( ) branch (we denote this class by C).

2) A weak approach occurs for $\alpha_1 > 0$ on the ascending branch (subclass C) and for $\alpha_1 < 0$ on the descending branch (subclass C). The type of branch after the approach does not change. (The classes are shown schematically in Fig. 1. All trajectories pass around m_2 clockwise.)

For large ΔV_1 and $|\alpha_1| < \pi/2$, only flybys of C exist. As ΔV_1 decreases, flybys of the subclass C appear. At $\Delta V_1 = 0$, flybys of the subclass C appear. With a sufficient decrease of ΔV_1, a flyby becomes impossible. The flybys with $\alpha_1 > 0$ disappear first, then those with $\alpha_1 < 0$. The solutions C , C for practically all V_1 pass outside the lunar disk, while the solutions C do so only for V_1 close to the minimal values.

In addition to flyby trajectories, in the plane ξ, η there may be approach trajectories that do not encompass the Moon but nevertheless allow one to look at its far side and return to m_1. They pass around m_2 counterclockwise. We shall call them "near-flyby" trajectories.

The near-flyby trajectories are also divided into two classes, analogous to the flyby ones:

1) A close approach occurs on the descending branch, which immediately passes into the ascending branch and only then returns to the Earth (class D).
2) A weak approach occurs either on the ascending branch (subclass D) or on the descending branch (subclass D). The type of branch after the approach does not change.

The evolution of near-flyby trajectories is analogous to the evolution of flyby trajectories.

3. **The special problem of flying around the Moon.** This is the problem of finding flybys in which the rocket returns to the Earth' s atmosphere. Such flybys are practically the most interesting. The method of solving problem 2 is applicable to this problem as well. But in it the miss is already a linear function of the errors in the initial data, which, together with the relative thinness of the atmosphere, leads to much stricter accuracy requirements. To each class of problem 2 there correspond two classes of problem 3, passing around m_2 in the same direction, and m_1 in different ones. We shall call them C_+ and C_- for the first class, and C_- and C_+ for the second class. Each pair bounds, in its class, the flybys of problem 2 corresponding to permissible dispersions of the initial data. In this case there also exist trivial limiting solutions for the second class that do not correspond to an approach to the Moon. Analogous facts also hold for the near-flyby classes.

The evolution of the classes of problem 3 follows easily from the evolution of the corresponding classes of problem 2.

The influence of scatter in the initial data depends not only on the character of the passage of the trajectory relative to m_1, but also on ρ. For small ρ the scatter has a stronger effect than for large ρ.

Example. For a flyby of type C^{∞}_{H+} with $\Delta V_1 = -0.083773$ and a flight time of 823,600 sec, it turned out that $\rho = 27$ thousand km. Errors in the initial data: in speed $\delta V_1 = 2 \cdot 10^{-4}$ km/sec, and in its direction $\delta\alpha_1 = 5 \cdot 10^{-3}$, cause, on return, changes in altitude of, respectively, 160 and 191 km, i.e., they are inadmissible (changes in altitude must not exceed tens of kilometers). Reducing ρ by an order of magnitude increases the influence of errors by 1-2 orders.

4. **The problem of a periodic flyby.** The simplest periodic solutions (solutions which, after approaching m_2, go toward m_1) are symmetric and constitute two one-parameter families—flyby and flight-around. By a

method analogous to the method of the Copenhagen school [(6)], a flyby solution concerning the atmosphere was found, but it turned out to pass inside the disk of the Moon and to be unstable. All solutions passing above the surface of the Moon are farther from m_1 by more than 94.8 thousand km (see dashed curve C in Fig. 1) and are also unstable. For example, a solution with $r_1 = 82824$ km, $\rho \simeq 1500$ km, with an error in speed $\delta V_1 \sim 0.001$ m/sec, escapes to infinity already from the 4th revolution. Errors in the initial data: in the initial speed $\delta V_1 = 10^{-4}$ km/sec, in its direction $\delta\alpha_1 = 10^{-4}$, and in altitude $\delta r_1 = 5$ km, cause at the end of the revolution changes in altitude of, respectively, 3471; 5252; 262 km.

5. **Classification of trajectories in the plane of the Moon's orbit and the problem of rocket acceleration.** In addition to the found impact and flyby trajectories of approach to m_2, there also exist approach trajectories corresponding either to acceleration of the rocket after approach, or to its deceleration (relative to m_1), and no other approach trajectories exist. Ascending deceleration trajectories go around the Moon clockwise, and acceleration trajectories counterclockwise. The maximum acceleration $\Delta V_{\max}$ for any V_1 and α_1 corresponds to a trajectory tangent to the disk of the Moon (cf. [(4)]). The maximum speed can be obtained in any direction. Since the plane of the Moon' s orbit makes a small angle with the planes of the orbits of the other planets, acceleration without expenditure of fuel can be used for interplanetary flight. For the problem of acceleration a method was created which, for any V_1 and α_1, makes it possible to obtain a trajectory with $\Delta V_{\max}$. It is easy to see that the solution of the problem of any acceleration, after replacing t by $-t$, always gives the solution of the problem of the same braking of the rocket when moving along the trajectory mirror-symmetric to the acceleration trajectory. This can be used, for example, when returning the rocket from interplanetary flight.

Remarks. 1. The question of the minimum speeds $V_{\min}$ is of fundamental importance (for $V_1 < V_{\min}$ our problems have no solution). It was solved earlier for other problems. By considering the surface of zero velocity passing through the libration point L_1, for the three-dimensional problem it was proved that $V_{\min}$ is one and the same in problems 1 and 2 and is equal to $V^*_{\min} = 10.8315$ km/sec practically for all points of the sphere $r_1 = 6571.118$ km. However, for V_1 close to $V^*_{\min}$, approach, i.e., entry of the rocket into the sphere of action of m_2, $\rho < \rho^*$, is possible only after a sufficient number of its revolutions (of the order of hundreds) around m_1. For approach to m_2 on the first revolution (which we assumed), the minimum speeds are different in problems 1-5 and exceed 10.9052 km/sec.

Along the way, the erroneousness of one theorem of Martin [(7)] was discovered. If one sets $m_2 = \mu$, $m_1 = 1 - \mu$, then for the distances $\rho_1(\mu)$ and $\rho_2(\mu)$ of the points L_1 and L_2 from μ, for $0 < \mu < 1$ one always has $\rho_1(\mu) < \rho_2(\mu)$, and their equality for $0 < \mu < 1$ is impossible, contrary to Martin' s theorem.

2. The question of the possibility of capture by the Moon of a rocket launched from the Earth

is evidently important for solving problems 1-5. By a method analogous to that applied by V. G. Fesenkov [8], it was shown for the three-dimensional problem that capture is impossible for any initial data. Along the way, the criterion for the possibility of capture (V. G. Fesenkov' s),

$$\frac{1}{a_2} + 2\sqrt{p_2}\cos i_2 < \mu F,$$

was corrected, which led to the criterion

$$\frac{\mu}{a_2} + 2\sqrt{\mu p_2}\cos i_2 < \mu F_1 + 10\left(\frac{\mu}{3}\right)^{2/3},$$

where a_2, p_2, i_2 are the elements of the rocket' s m_2-centric orbit, and F and F_1 are finite.

3. The approximate theoretical methods used were based on neglecting the perturbations of m_1 for $\rho < \rho^* \simeq 66\,100$ and the perturbations of m_2 for $\rho > \rho^*$; some other secondary factors were also neglected. However, almost all results were checked and invariably confirmed by a more accurate method—numerical integration. The integration was carried out for regularized equations in Thiele variables on the electronic computer TsDM [9], with 5-7 significant digits. Iterative methods were used to find trajectories of various purposes; in all, more than 600 trajectories were computed.

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Note: Figure translations are in progress. See original paper for figures.