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Abstract

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V. N. ROGINSKII

EQUIVALENT TRANSFORMATIONS OF RELAY CIRCUITS OF CLASS Π

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1. Modern theory of relay-contact circuits considers mainly transformations of contact networks. Transformations of relay circuits, i.e., circuits that include not only contacts but also relay windings and resistances having finite conductance values, have so far been studied very little. Certain considerations concerning the transformation of normal or inverse relay circuits* are given in works ([1]–[5]).

Relay circuits in which, for the same character and order of external action, the relays operate in the same order and the actuating circuits are in the same state are called **equivalent** ([2]).

In the present note we set forth principles for transformations of relay circuits of class Π that lead to equivalent circuits. The application of these transformations expands the range of circuits that can be synthesized, makes it possible to create mixed circuits and, in particular, permits simplification of multirelay circuits with such a calculation that each of the contacts acts, as far as possible, on a larger number of relays.

2. In contrast to contacts, which have two limiting conductance values: 0 (open circuit) and 1 (short circuit), elements G (relay windings, resistances) have finite conductance, i.e., satisfy the inequality

$$0 < G < 1. \tag{1}$$

An element G of finite conductance, introduced into the circuit of some relay A, depending on the connection scheme, the applied voltage, and the parameters of the relay and of the element G, may either not affect the operation of relay A, or may completely disrupt its operation. Let us introduce the concept of the **order of conductance**, characterizing the effect of the conductance G on the operation of relay A. If the conductance G, connected in series with relay A, leads to such a decrease of current in the winding that the relay does not hold, while parallel connection of this same conductance does not disturb the operation of the relay, then we shall say that the order of the conductance G is **less than** the order of the conductance A, and denote this by the symbol

$G < A$. Analogously, we shall regard the conductance as being of **greater order** ($G > A$) if, when connected in parallel with winding A, it shunts the latter and disrupts the operation of relay A. If, however, both parallel and series connection of the conductance G have no effect on the operation of relay A, then we shall say that

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* A normal relay circuit is a circuit in which the contact network acting on the relay is connected in series with the winding of this relay, and an inverse one is a circuit in which the contact network is connected in parallel.

conductance G and A have the same order (we shall denote this by $G_{=A}$, or simply G). The concept of the order of conductance may be illustrated by Table 1.

Let us also agree that the parameters of individual relays and other elements of the circuit are chosen so that: a) if $A_{=}$, then ${}_{=A}$; b) if $A_{>}$, then ${}_{<A}$, and conversely; c) the parallel or series connection of a finite number of conductances of one order leads to a conductance of the same order.

Table 1

Element G is connected to the relay-A coil	Relay A operates	Relay A does not operate
	Order of conductance G	Order of conductance G
In series	$G_{>A}$, G, $G_{\geq A}$	$G_{<A}$
In parallel	$G_{<A}$, G, $G_{\leq A}$	$G_{>A}$

When an expression or a part of it contains elements of one order of conductance, we shall write them without any indices. For example, the notation $(A+)_{<B}$ will mean that A and have the same order of conductance and that the conductance of each of them, as well as their total conductance, has an order lower than the conductance of B.

3. From the definition of the order of conductance the following properties follow: a) if $A_{>}$ and ${}_{>B}$, then $A_{>B}$; b) if $A_{\geq}$, and ${}_{\geq A}$, then the conductances A and are of one order; c) if $A_{>}$, and ${}_{>A}$, then the circuit is unrealizable; d) with a series connection of conductances of different orders, the total conductance is determined by the conductance of the lower order; e) with a parallel connection of conductances of different orders, the total conductance is determined by the conductance of the higher order.

4. Transformations that lead to the obtaining of equivalent circuits will be called **equivalences** and denoted by the equality sign (=). If, in addition, the conductance is also preserved (to within the order), then we shall speak of **absolute equivalence** and denote it by the identity sign (≡).

For relay circuits the following fundamental laws are valid ($x, y, z, ...$ are contact chains; $A, , B, ...$ are relay windings; G is an element of finite conductance):

a) commutative ([1,3]):

$$xA \equiv Ax; \qquad x + A \equiv A + x; \tag{2}$$

b) associative ([1,3]):

$$xyA \equiv (xy)A \equiv x(yA); \qquad x + y + A \equiv (x + y) + A \equiv x + (y + A); \tag{3}$$

c) distributive ([1,3]):

$$x(A +) \equiv xA + x; \qquad x + A \equiv (x + A)(x +). \tag{4}$$

In addition, for relay circuits without opposing windings, under the condition that for the operation of a relay it is sufficient for current to pass through at least one relay winding, the following laws are valid ($A^1, A^2, ...$ are the windings of relay A, numbered in any order):

d) repetition:

$$A = A^1 + A^2 + ... ; \qquad A = A^1 \cdot A^2 ... ; \tag{5}$$

e) distributive:

$$(x + y)A = xA^1 + yA^2; \qquad xy + A = (x + A^1)(y + A^2) \tag{6}$$

(equivalences (5) and (6) will be absolute if all finite conductances have the same order);

e) decompositions ([3,5]) (under the condition that all coils $A^1, A^2, ...$ of each relay have the same order as the relay A in the original circuit):

$$\begin{aligned} &F(x, y, \dots, G, A, B, ...) = \\ &\quad = xF(1, y, \dots, G, A^1, B^1, ...) + \bar{x}F(0, y, \dots, G, A^2, B^2, ...) = \\ &\quad = xyF(1, 1, \dots, G, A^1, B^1, ...) + x\bar{y}F(1, 0, \dots, G, A^2, B^2, ...) + \\ &\qquad + \bar{x}yF(0, 1, \dots, G, A^3, B^3, ...) + \bar{x}\bar{y}F(0, 0, \dots, G, A^4, B^4, ...); \end{aligned} \tag{7}$$

$$\begin{aligned} &F(x, y, \dots, G, A, B, ...) = \\ &\quad = [x + F(0, y, \dots, G, A^1, B^1, ...)][\bar{x} + F(1, y, \dots, G, A^2, B^2, ...)] = \\ &\quad = [x + y + F(0, 0, \dots, G, A^1, B^1, ...)][x + \bar{y} + F(0, 1, \dots, G, A^2, B^2, ...)] \times \\ &\qquad \times [\bar{x} + y + F(1, 0, \dots, G, A^3, B^3, ...)][\bar{x} + \bar{y} + F(1, 1, \dots, G, A^4, B^4, ...)]. \end{aligned}$$

For circuits consisting of elements of finite conductivity (coils) of one order of conductivity, all of them may be connected in any combination in class Π, i.e., the equivalence holds

$$AB \equiv A + B. \tag{8}$$

5. Elements of finite conductivity may be introduced into relay circuits (X is any circuit):

$$xA \equiv xG_{\geq A}A \equiv x(G_{\geq A} + X)A; \qquad xA = (x + G_{<A})A = (x + XG_{<A})A; \tag{9}$$

$$y + A = yG_{>A} + A = y(G_{>A} + X) + A;$$

$$y + A \equiv y + A + G_{\leq A} \equiv y + A + XG_{\leq A}. \tag{10}$$

Contact circuits may likewise be introduced into relay circuits in accordance with the general rules for the "extension" of contact circuits ([2,5,7]), or on the basis of the following equivalences:

$$xA = xA + \frac{\bar{x}}{0}^{*}; \qquad x + A = (x + A)\frac{\bar{x}}{1}. \tag{11}$$

By introducing elements of finite conductivity one may change the conductivity of a circuit; in particular, any relay circuit may be transformed so that it will possess the property of an element of finite conductivity, i.e., satisfy inequality (1):

$$xA = xA + \frac{\bar{x}}{1}G; \qquad x + A = (x + A)\left(\frac{\bar{x}}{0} + G\right). \tag{12}$$

6. An equivalent transformation is the **inversion** of a relay circuit ([2,8]), whereby the structural conductivity becomes the inverse one. For circuits with conductivities of different orders, we shall agree that under inversion the order of conductivity is changed to the inverse, i.e.,

$$\bar{A}_{>B} = A_{<B}; \qquad \bar{A}_{\geq B} = A_{\leq B}; \qquad \bar{A}_{<B} = A_{>B}; \qquad \bar{A}_{\leq B} = A_{\geq B}. \tag{13}$$

In this case inversion of relay circuits may be carried out according to the inversion laws of the algebra of contact circuits.

It is possible to invert not the whole circuit, but parts of it. **If in a relay circuit one can single out a relay two-terminal network Z,**

* The symbol $\frac{x}{y}$, called an equivalence [5], is introduced for writing indeterminate solutions and indicates that under the given conditions either of the expressions x or y may be taken. In special cases the symbol $\frac{b}{0}$ means that $0 \leq \frac{b}{0} \leq b$, and the symbol $\frac{a}{1}$ means that $a \leq \frac{a}{1} \leq 1$, i.e., these symbols are equivalent to another notation for the concept of inclusion in the algebra of logic [6]. For some operations with the expressions $\frac{b}{0}$ and $\frac{a}{1}$, see [5,7].

into which the windings of all relays of the circuit will enter, then either the circuit of only this two-terminal network or the entire circuit can be inverted, leaving Z unchanged.

Circuits of class H can also be inverted, using, for example, the graphical method [2].

7. In circuits with valve elements, the latter may be regarded as elements of finite conductance possessing a conductance whose order in the open state is greater than the order of the conductance of all relays, and in the closed state is smaller.

8. To combine individual circuits of multi-relay schemes, it is necessary to bring the circuits of the individual windings to the same structure by equivalent transformations and by introducing elements of finite conductance and contact circuits, after which these circuits are "superposed," with the introduced circuits replaced by windings and contacts from other circuits.

Special cases of combination are the known methods of parallel connection of normal circuits or series connection of inverse circuits with windings of identical conductance orders.

9. An example of the synthesis of a binary-counter circuit, whose structural formula in normal form [2,5] is:

$$F = (u\bar{b} + \bar{u}a)A + (\bar{u}a + ub)B = u\bar{b}A^1 + ubB^1 + \bar{u}aA^2B^2.$$

Transform the individual circuits:

$$F_1 = u\bar{b}A^1 = u(b + A^1) = u(bG_{>A^1} + A^1) = (u + xG_{<A^1})(bG_{>A^1} + A^1);$$
$$F_2 = ubB^1 = u(bB^1 + G_{<B^1}) = (u + xG_{<B^1})(bB^1 + G_{\leq B^1});$$
$$F_3 = \bar{u}aA^2B^2 = u + aA^2B^2 = (u + aA^2B^2)G_{>A^2,B^2}.$$

Combining the obtained expressions, which have the same structure, we obtain a new structural formula:

$$F = [u + a(A^2B^2)_{<A^1}](bB^1_{>A^1} + A^1).$$

Other structures can also be obtained.

The corresponding circuit has one contact on each relay, and consequently it is impossible to create a circuit with a smaller number of contacts. In calculating the parameters of the relays of the circuit, it is necessary to take into account the requirements that follow from the concept of the order of conductance.

Laboratory for the Development of Scientific
Problems of Wire Communication
Academy of Sciences of the USSR

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CITED LITERATURE

1. V. I. Shestakov, *ZhTF*, **2**, issue 6, 532 (1941).
2. M. A. Gavrilov, *Theory of Relay-Contact Circuits*, M.–L., 1950.
3. J. Shekel, *Proc. Inst. Rad. Eng.*, **41**, No. 7, 913 (1953).
4. G. Schliebs, *Funk u. Ton.*, **8**, No. 2, 57 (1954).
5. V. N. Roginsky, A. D. Kharkevich, *Relay Circuits in Telephony*, 1955.
6. L. Couturat, *Algebra of Logic*, Odessa, 1909.
7. V. N. Roginsky, *Collection of Scientific Works on Wire Communication*, Publishing House of the Academy of Sciences of the USSR, issue 4, 111, 1955.
8. M. A. Gavrilov, V. A. Khvoshch, *DAN*, **75**, No. 5, 685 (1950).

Note: Figure translations are in progress. See original paper for figures.