GEOPHYSICS

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Abstract

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PRECOMPUTATION OF THE WIND FIELD AND THE METEOROLOGICAL ELEMENTS ASSOCIATED WITH IT

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The methods currently used for calculating changes in the geopotential field with time, as a rule, rely on replacing the actual wind by the geostrophic wind. For precomputing the wind field, however, it is essential to take into account the deviations of the wind from the geostrophic values, especially in the upper troposphere. Using the smallness of the parameter

$$\varepsilon = \frac{1}{l}\frac{\partial v_s}{\partial s} \tag{1}$$

in the case of a large-scale process (l is the Coriolis parameter, $\partial v_s/\partial s$ is the characteristic value of the horizontal derivative of the horizontal component of the wind velocity), one may represent the components of the deviation of the wind from the geostrophic wind, u', v', along the axes x, y in the form (1)

$$u' = -\frac{1}{l}\left(\frac{\partial v_\mathrm{r}}{\partial t} + u_\mathrm{r}\frac{\partial v_\mathrm{r}}{\partial x} + v_\mathrm{r}\frac{\partial v_\mathrm{r}}{\partial y}\right);$$

$$v' = \frac{1}{l}\left(\frac{\partial u_\mathrm{r}}{\partial t} + u_\mathrm{r}\frac{\partial u_\mathrm{r}}{\partial x} + v_\mathrm{r}\frac{\partial u_\mathrm{r}}{\partial y}\right). \tag{2}$$

Here t is time; the x-axis is directed eastward, the y-axis northward; $u_\mathrm{r}, v_\mathrm{r}$ are the components of the geostrophic wind, related to the geopotential field Φ by the relations

$$u_\mathrm{r} = -\frac{1}{l}\frac{\partial \Phi}{\partial y}, \qquad v_\mathrm{r} = \frac{1}{l}\frac{\partial \Phi}{\partial x}. \tag{2a}$$

Equations (2) and (2a) replace the equations of horizontal motion, in which terms of order ε^2 relative to the leading terms have been discarded. The remaining equations of the dynamics of large-scale atmospheric processes in the free atmosphere have the form (see, for example, (2), Ch. X)

$$RT = -\zeta \frac{\partial \Phi}{\partial \zeta}; \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \widetilde{w}}{\partial \zeta} = 0; \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - \frac{(\gamma_a - \gamma)RT}{g\zeta}\,\widetilde{w} = 0. \tag{5}$$

Here ζ is the ratio of the air pressure to the standard pressure (1000 mb); T is the absolute temperature of the air; R is the gas constant; $\gamma_a - \gamma$ is the difference between the adiabatic and actual vertical temperature gradients; g is the acceleration due to gravity. The motion is assumed adiabatic.

The quantity $\widetilde{w}$ is an analogue of the vertical velocity of the air w and is determined by the relation

$$\widetilde{w} = -\frac{\zeta}{RT}\left(gw - \frac{\partial \Phi}{\partial t} - u\frac{\partial \Phi}{\partial x} - v\frac{\partial \Phi}{\partial y}\right). \tag{6}$$

The absence of mass flux through the upper boundary of the atmosphere is described by the boundary condition

$$\widetilde{w} = 0 \quad \text{for } \zeta = 0. \tag{7}$$

Near the surface of the Earth, $\widetilde{w}$ is determined mainly by the orography of the terrain and turbulent friction. Given the field of the geostrophic wind determined up to and including the instant t_1, we write the boundary condition in the following form:

$$\widetilde{w} = \widetilde{w}_1(x, y, t) \quad \text{for } \zeta = 1 - \delta,\ t \le t_1, \tag{8}$$

where $\widetilde{w}_1(x, y, t)$ is a known function of its arguments; in an approximate calculation δ may be regarded as constant, $\delta \approx 0.05$.

Substituting in equation (4)

$$u = -\frac{1}{l}\frac{\partial \Phi}{\partial y} + u', \qquad v = \frac{1}{l}\frac{\partial \Phi}{\partial x} + v',$$

we obtain

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial \widetilde{w}}{\partial \zeta} = \frac{1}{l}\frac{dl}{dy}\frac{\partial \Phi}{\partial x}. \tag{4a}$$

In the subsequent calculations l, dl/dy, and the quantity

$$a^2 = \frac{(\gamma_a - \gamma) R^2 T}{g l^2} \tag{9}$$

will be regarded as constant. The relative error of the calculations in this case will not exceed ε (3). With the same accuracy, u, v in equation (5) may be replaced by the quantities u_r, v_r:

$$a^2 \widetilde{w} = \frac{R\zeta}{l^2} \left(\frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial x} + v_r \frac{\partial T}{\partial y} \right). \tag{5a}$$

Let us now form quantities analogous to the components of the vorticity vector u'/a, v'/a, $\widetilde{w}$. We have:

$$\begin{aligned} a^2 \frac{\partial \widetilde{w}}{\partial y} - \zeta^2 \frac{\partial v'}{\partial \zeta} &= -\frac{2\zeta^2}{l^3} \{\Phi_y, \Phi_\zeta\}; \\ \zeta^2 \frac{\partial u'}{\partial \zeta} - a^2 \frac{\partial \widetilde{w}}{\partial x} &= -\frac{2\zeta^2}{l^3} \{\Phi_\zeta, \Phi_x\}; \\ a^2 \frac{\partial v'}{\partial x} - a^2 \frac{\partial u'}{\partial y} &= -\frac{2a^2}{l^3} \{\Phi_x, \Phi_y\}. \end{aligned} \tag{10}$$

Here the symbol $\{A, B\}$ denotes the functional determinant

$$\{A, B\} = \frac{\partial A}{\partial x} \frac{\partial B}{\partial y} - \frac{\partial A}{\partial y} \frac{\partial B}{\partial x}. \tag{11}$$

Since the right-hand sides of (10) and (4a) contain no time derivatives, the fields u', v', $\widetilde{w}$ can be determined from the geostrophic field

of the wind at the initial time t_0. For this purpose it is convenient to introduce new functions $\psi_1, \psi_2, \psi_3, \chi$ by the relations

$$\begin{aligned} u_1 &= \frac{\partial \psi_3}{\partial y} - \frac{\partial \psi_2}{\partial \zeta} + a^2 \frac{\partial \chi}{\partial x}; \\ v' &= \frac{\partial \psi_1}{\partial \zeta} - \frac{\partial \psi_3}{\partial x} + a^2 \frac{\partial \chi}{\partial y}; \\ \widetilde{w} &= \frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_1}{\partial y} + \zeta^2 \frac{\partial \chi}{\partial \zeta} \end{aligned} \tag{12}$$

under the additional condition

$$a^2 \frac{\partial \psi_1}{\partial x} + a^2 \frac{\partial \psi_2}{\partial y} + \zeta^2 \frac{\partial \psi_3}{\partial \zeta} = 0. \tag{13}$$

Then from equations (10) and (4a) we obtain

$$\begin{aligned} \zeta^2 \frac{\partial^2 \psi_1}{\partial \zeta^2} + a^2 \Delta \psi_1 &= \frac{2\zeta^2}{l^3} \{\Phi_y, \Phi_\zeta\}; \\ \zeta^2 \frac{\partial^2 \psi_2}{\partial \zeta^2} + a^2 \Delta \psi_2 &= \frac{2\zeta^2}{l^3} \{\Phi_\zeta, \Phi_x\}; \\ \frac{\partial}{\partial \zeta} \left(\zeta^2 \frac{\partial \psi_3}{\partial \zeta} \right) + a^2 \Delta \psi_3 &= \frac{2a^2}{l^3} \{\Phi_x, \Phi_y\}; \\ \frac{\partial}{\partial \zeta} \left(\zeta^2 \frac{\partial \chi}{\partial \zeta} \right) + a^2 \Delta \chi &= -\frac{1}{l^2} \frac{dl}{dy} \Phi_x. \end{aligned} \tag{14}$$

Here Δ is the Laplace operator in the plane, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

The boundary conditions corresponding to (7) and (8) may, in particular, be specified in the following simple way:

$$\begin{aligned} &\text{for } \zeta = 0 \qquad \psi_1 = \psi_2 = \frac{\partial \psi_3}{\partial \zeta} = \frac{\partial \chi}{\partial \zeta} = 0; \\ &\text{for } \zeta = 1 - \delta \qquad \psi_1 = \psi_2 = \frac{\partial \psi_3}{\partial \zeta} = 0, \quad \frac{\partial \chi}{\partial \zeta} = \frac{\widetilde{w}_1(x, y, t)}{(1-\delta)^2}. \end{aligned} \tag{15}$$

Equations (14) may be integrated numerically, analogously to ([4]), or analytically; after this u', v', $\widetilde{w}$ are found from (12), and the changes in the geostrophic wind and temperature with time from (2) and (5a).

Thus, we can construct the fields of the indicated meteorological elements for the time $t_0 + \tau$. A numerical forecast of the fields of geostrophic and actual wind, vertical velocity, and air temperature is obtained by repeating the process the required number of times.

The values of the quantities u', v' may also be used to determine the velocity of displacement of singular points in the geopotential field. For example, the components c_x, c_y of the velocity of displacement of a baric center (i.e., the point at which $\partial \Phi / \partial x = \partial \Phi / \partial y = 0$) are found from the equations

$$\left(\frac{\partial}{\partial t} + c_x \frac{\partial}{\partial x} + c_y \frac{\partial}{\partial y} \right) \frac{\partial \Phi}{\partial x} = 0;$$

$$\left(\frac{\partial}{\partial t} + c_x \frac{\partial}{\partial x} + c_y \frac{\partial}{\partial y} \right) \frac{\partial \Phi}{\partial y} = 0,$$

whence, taking account of (2) and $u_\mathrm{g} = 0$, $v_\mathrm{g} = 0$, we obtain

$$c_x = l^2 \frac{u'\Phi_{yy} - v'\Phi_{xy}}{\{\Phi_x, \Phi_y\}}, \qquad c_y = l^2 \frac{v'\Phi_{xx} - u'\Phi_{xy}}{\{\Phi_x, \Phi_y\}}. \tag{16}$$

The method of numerical forecasting proposed here has a number of advantages not only over the "barotropic model" widely used in practice, but also over the baroclinic model (see, for example, (4)).

First of all, it must be borne in mind that the "individual" changes over time in wind and air temperature (in horizontal motion with geostrophic velocity), in the usual forecasting method, prove, as a rule, to be small differences between two large quantities—local and convective changes—which substantially affects the accuracy of determining these very important elements. The proposed method, in which the individual changes of meteorological elements are found directly from (2) and (5a), is free of this shortcoming. In addition, it is quite important that the right-hand sides of equations (14) contain only second spatial derivatives of the geopotential, whereas an entirely analogous operator of $d\Phi/dt$ is equal to a right-hand side containing higher (third) derivatives. Since the computation of third derivatives of empirically determined fields is associated with large errors, finding the geostrophic wind and, still more, the deviations of the wind from the geostrophic wind by the usual method is in principle less accurate than by the direct method set forth above.

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Note: Figure translations are in progress. See original paper for figures.