MECHANICS

D. I. MANZHERON

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Abstract

Full Text

MECHANICS

D. I. MANZHERON

ON REDUCED ACCELERATIONS OF ARBITRARY ORDER AND SOME OF THEIR EXTREMAL PROPERTIES

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Let $\mathbf{w}_M^{(n)}$ be the acceleration of order n of a point M of a rigid body undergoing plane-parallel motion, expressed by Somov' s generalized formulas ([1]), written in matrix notation:

$$\begin{Bmatrix} x_1^{(n+1)} \\ x_2^{(n+1)} \end{Bmatrix} = \begin{Bmatrix} x_{10}^{(n+1)} \\ x_{20}^{(n+1)} \end{Bmatrix} + \begin{Bmatrix} -A_n & -B_n \\ B_n & -A_n \end{Bmatrix} \begin{Bmatrix} x_1 - x_{10} \\ x_2 - x_{20} \end{Bmatrix}, \tag{1}$$

where $A = A_n(t)$ and $B_n = B_n(t)$ are determined by the recurrence formulas

$$A_{n+1} = \dot{A}_n + \dot{\theta} B_n, \qquad B_{n+1} = \dot{B}_n - \dot{\theta} A_n, \qquad A_1 = \dot{\theta}^2, \qquad B_1 = \ddot{\theta}, \tag{2}$$

$$\left(\cdot \equiv \frac{d}{dt} \right), \qquad x_i^{(n+1)} = \frac{d}{dt} x_i^{(n)} \quad (i = 1, 2;\ n = 1, 2, ...)$$

and $\theta = \theta(t)$ is the angle of rotation of the body.

Theorem 1. *The locus of points M^*, determined by the vector equation*

$$\mathbf{r}_{M^*} = \mathbf{r}_M + \lambda_n \mathbf{w}_M^{(n)}, \tag{3}$$

where M is a point lying on some line (D_M) taking part in plane-parallel motion, is a line (D_{M^*}) forming with the line (D_M) an angle φ_n determined by the relation

$$\operatorname{tg} \varphi_n = \frac{\lambda_n B_n}{1 - \lambda_n A_n}. \tag{4}$$

It is obvious that the angle φ_n depends only on the state of motion of the link under consideration at the given instant of time and on the chosen value of the parameter λ_n.

Theorem 2. *The reduced accelerations of order n, introduced in* ([2]) *and determined by the relations*

$$\mathbf{w}_r^{(n)} = \frac{\mathbf{w}^{(n)}}{A_n}, \qquad \mathbf{w}_r^{(1)} = \frac{\mathbf{w}^{(1)}}{A_1} \equiv \frac{\mathbf{w}}{A_1} \quad (n = 1, 2, ...), \tag{5}$$

where A_n (and B_n) are expressed by the recurrence formulas (2), are characterized by the extremal property of the function (4) of the angle φ_n.

Theorem 3 (on the distribution of accelerations of arbitrary order). *The locus of the endpoints of the reduced accelerations of order n of points of a line (D) undergoing plane-parallel motion is a line (D_r) perpendicular to the given one.*

Theorem 4 (generalized Kotelnikov theorem ([3])). *Circles whose diameters are the reduced accelerations of order n of points of some rigid body undergoing plane-parallel motion, pro-*

pass through the instantaneous center of accelerations of the same order $P_n(x_{1P_n}, x_{2P_n})$, given in matrix notation by the following equality:

$$\begin{Bmatrix} x_{1P_n} \\ x_{2P_n} \end{Bmatrix} = \begin{Bmatrix} x_{10} \\ x_{20} \end{Bmatrix} + \frac{1}{A_n^2 + B_n^2} \begin{Bmatrix} A_n & -B_n \\ B_n & A_n \end{Bmatrix} \begin{Bmatrix} x_{10}^{(n+1)} \\ x_{20}^{(n+1)} \end{Bmatrix} \quad (n = 1, 2, ...). \tag{6}$$

Theorem 5 (generalized theorem of similarity). The endpoints of the accelerations of the n-th order of the points of a rigid body undergoing plane-parallel motion, in the case when the origins of these accelerations are placed at one point, form a figure similar to this rigid body, rotated through the angle

$$\theta_n = \operatorname{arc\,tg} \frac{B_n}{A_n}, \tag{7}$$

where A_n and B_n are determined by relations (2).

The theorems established serve as the basis for a number of theorems ([7],[8]) and lead to a new method, called the **method of reduced accelerations of any order**, developed by a collective under the author’ s direction. The method of reduced accelerations of any order makes it possible, proceeding from the known classifications of I. I. Artobolevsky ([4]), G. G. Baranov ([5]), and others, and by studying the most characteristic Assur groups, to carry out a very simple systematic investigation, by a graphico-analytical method, of the classical problem of the distribution of first-order accelerations for all Assur groups belonging to plane mechanisms.

Iași Polytechnic Institute
Iași, Romanian People’ s Republic

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Note: Figure translations are in progress. See original paper for figures.