HOMING AS A PROBLEM OF TECHNICAL CYBERNETICS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

Mechanics

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HOMING AS A PROBLEM OF TECHNICAL CYBERNETICS

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In works devoted to the homing of guided projectiles (2, 3), the authors confined themselves to studying the kinematics of a projectile regarded as a point. The possibility of realizing trajectories was then determined by the magnitude of the normal acceleration or by the magnitude of the angular velocity of the line of sight connecting the projectile and the target.

In the monograph (4), the author assumes that the rudder deflection angle is proportional to the angular velocity of rotation of the velocity vector and studies the dynamics of projectile control, using the value of this angular velocity obtained from a kinematic consideration of the problem. In doing so, the dynamics of the projectile itself as a rigid body with a control rudder is not taken into account, which, for some relations of the parameters, leads to results directly contrary to the truth, and in other cases to significant errors.

On the other hand, the appearance of modern high-speed computing machines has led to a tendency to compute nearby perturbed projectile trajectories separately, abandoning consideration of the system of differential equations in variations. With such an approach to the choice of the principal parameters of the automatic-control system, nothing can be said about the stability of motion over a finite time interval.

Qian Xuesen (1) considered, as a problem of technical cybernetics, the stabilization of the flight range of a ballistic rocket. Using a system of differential equations in variations, Qian Xuesen determines the choice of the automatic-control system for the motion on the basis of the conditions for reducing the miss. We shall apply this principle to the analysis and synthesis of homing of guided projectiles.

Fig. 1

Dynamics of ideal motion. The system of differential equations of the projectile, the velocity vector of whose center of inertia is continuously directed toward the target (Fig. 1), will be (for motion in a horizontal plane and a small angle of attack)

$$mv(\dot{\varphi} - \dot{\alpha}) = (T + C_L v^2)\alpha; \qquad I_z \ddot{\varphi} = -k_1\beta - k_2\dot{\varphi} + k_3\alpha;$$

$$\dot{x} = -v\cos\psi, \qquad \dot{y} = -v\sin\psi; \qquad \varphi = \alpha + \psi; \qquad y_s = y_{s0} + v_{st}. \tag{1}$$

In these equations: m is the mass of the projectile; I_z is the moment of inertia of the projectile about an axis passing through the center of inertia perpendicular to the horizontal plane of pursuit; v is the speed of the center of inertia of the projec-

row, constant in magnitude; α is the angle of attack; φ is the angle of turn of the projectile; ψ is the angle between the sighting line and the perpendicular to the target trajectory; k_1, k_2, k_3 are constant coefficients; β is the rudder deflection angle; T is the thrust force, constant in magnitude; C_L is the lateral-force coefficient; x, y are the coordinates of the projectile' s center of inertia.

The last equation describes the rectilinear and uniform motion of the target. The solution of this system is the law for the rudder deflection angle

$$\beta = A_1\left[\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\psi + \operatorname{tg}\psi}\right]^{1/k}\cos^2\psi + A_2\left[\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\psi + \operatorname{tg}\psi}\right]^{2/k}\cos^3\psi\left(2\sin\psi + \frac{1}{k}\right) + \\ + A_3\left[\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\psi + \operatorname{tg}\psi}\right]^{3/k}\cos^4\psi\left[8\sin^2\psi + \frac{7}{k}\sin\psi + \frac{2-2k^2}{k^2}\right], \tag{2}$$

which ensures pursuit of the target along the ideal trajectory. The constants A_1, A_2, A_3 are expressed through the coefficients of the original equations, $k = v_s/v$. The dependence of the sighting-line angle on time is determined from the original equations

$$\frac{k}{2(1+k)}\left[(\operatorname{tg}\psi + \sec\psi)^{1/k+1} - (\operatorname{tg}\psi_0 + \sec\psi_0)^{1/k+1}\right] + \frac{k}{2(1-k)}\left[(\operatorname{tg}\psi + \sec\psi)^{1/k-1}\right. \\ \left. -(\operatorname{tg}\psi_0 + \sec\psi_0)^{1/k-1}\right] = -\frac{v_s}{x_0}\left[\operatorname{tg}\psi_0 + \sec\psi_0\right]^{1/k} t. \tag{3}$$

as well as the angular velocity of the sighting line and the angle of attack:

$$\dot{\psi} = -\frac{v_s}{x_0}\left[\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\psi + \operatorname{tg}\psi}\right]^{1/k}\cos^2\psi, \qquad \alpha = -\frac{mv\cdot v_s}{(T + C_L v^2)x_0}\left[\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\psi + \operatorname{tg}\psi}\right]^{1/k}\cos^2\psi. \tag{4}$$

The projectile will follow the ideal trajectory as long as the rudder deflection angle, determined by equation (2), does not exceed the limiting angle of turn permitted by the design. Exceeding the admissible angle β may occur both far from the target and at the end of the pursuit process, near the target. In the first case the initial conditions are decisive: the distance between the projectile and the target a_0, and the initial angle ψ_0. Of greatest interest is the behavior of the rudder deflection angle near the target. Here the only parameter determining the character of the change in the rudder deflection angle is the ratio of the velocities of the target and the projectile—the quantity k. At the moment the target is hit, $\psi = \pi/2$. In this case the values of the angle β, the angular velocity of the sighting line $\dot{\psi}$, the angle of attack α, and the normal acceleration of the projectile' s center of inertia become indeterminate; resolving this indeterminacy, we find that the angle β near the target increases without bound if $k < 3/4$, and tends to zero if $k > 3/4$.* The angular velocity of the sighting line, the angle of attack, and the normal acceleration increase without bound when $k < 1/2$ and tend to zero when $k > 1/2$. Thus, for values $1/2 < k < 3/4$ the correlation between these quantities near the target is completely disrupted. The feasibility of pursuit can be determined only from consideration of the rudder deflection angle, and not of the angular velocity of the sighting line, the normal acceleration, the overload widely used in aviation, or the radius of circulation used in ship dynamics.

Dynamics of the real motion. The solution of the problem of the ideal motion of the projectile makes it possible to obtain a system of differential equations in variations, on the basis of which the parameters of the automatic-control system that ensure stable guidance of the projectile to the target are selected. Under real conditions the projectile is equipped with a homing head, whose axis is continuously directed at the target. In this case the angle $\Delta\psi_1$ between the direction toward the target and the velocity vector of the projectile' s center of inertia continuously changes (Fig. 2). The mismatch angle arises even in the absence of disturbances in the motion of the target and the projectile, since no commands are issued to the rudder according to equation (2).

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* For $k = 1/2;\ 2/3;\ 3/4$ the rudder deflection angle near the target tends to a finite limit.

Earlier [4] this angle $\Delta\psi_1$ was considered as the angle of mismatch, and the task of automatic control was to reduce this angle to zero. In contrast to this, we shall take as the angle of mismatch the angle $\Delta\psi$ (Fig. 2). This is the angle between the actual, real direction of the velocity vector and the sighting line drawn from the point B, where the projectile should have been in ideal motion. The angle of rudder deflection in real motion is effected according to the law

$$\begin{aligned}\beta_p = S_{10}\Delta\psi + S_{11}\Delta\dot{\psi} + \\ + S_{12}\Delta\ddot{\psi} + \ldots \equiv S\Delta\psi.\end{aligned} \qquad (5)$$

Fig. 2

Figure 2: Fig. 2

Fig. 2

Composing the differential equations of the real motion, which will be analogous to equations (1), and subtracting from them the equations of the ideal motion, we obtain a system of differential equations in variations

$$mv(\Delta\dot{\varphi} - \Delta\dot{\alpha}) = (T + C_L v^2)\Delta\alpha; \quad I_z \Delta\ddot{\varphi} = -k_2 \Delta\dot{\varphi} + k_3 \Delta\alpha - k_1[S\Delta\psi - \beta];$$

$$\Delta\varphi = \Delta\alpha + \Delta\psi; \quad \Delta\dot{x} = -\dot{y}\Delta\psi; \quad \Delta\dot{y} = \dot{x}\Delta\psi. \tag{6}$$

The first three differential equations in (6) are transformed into one linear nonhomogeneous differential equation of the 3rd order with constant coefficients. In dimensionless form it takes the form

$$(D^3 + m_1 D^2 + m_2 D + m_3)\Delta\psi = f(\tau), \tag{7}$$

where $\tau = t/T_1$; T_1 is the total time of the pursuit process; $D = d/d\tau$,

$$f(\tau) = \frac{(T + C_L v^2)k_1}{I_{zmv}} T_1^3 \beta = H\beta$$

(the rudder deflection angle β corresponds to (2)).

The constant coefficients in equation (7) are equal to:

$$m_1 = \left\{ \frac{k_2}{I_z} + \frac{T + C_L v^2}{mv} + \frac{k_1(T + C_L v^2)}{I_{zmv}} S_{12} \right\} T_1; \tag{8}$$

$$m_2 = \left\{ \frac{k_2(T + C_L v^2)}{I_{zmv}} - \frac{k_3}{I_z} + \frac{k_1(T + C_L v^2)}{I_{zmv}} S_{11} \right\} T_1^2; \qquad m_3 = \frac{k_1(T + C_L v^2)}{I_{zmv}} T_1^3 S_{10}.$$

Thus, summarizing the content of the problem, we have differential equation (7), where the right-hand side is determined by the dependence of β on ψ (2), and the dependence $\psi = f_1(\tau)$ is expressed through (3). The pursuit process begins at $\tau = 0$ and ends at $\tau = 1$.

The solution of equation (7) consists of two parts: $\Delta\psi = (\Delta\psi)_0 + (\Delta\psi)_n$. The solution $(\Delta\psi)_0$ corresponds to nonhomogeneous initial conditions and $f(\tau) = 0$. The solution $(\Delta\psi)_n$ corresponds to homogeneous (zero) initial conditions and $f(\tau) \neq 0$.

Denoting the roots of the characteristic equation by r_1, r_2, r_3, we have

$$(\Delta\psi)_0 = \sum_{i=1}^{3} \frac{C_2 + (r_i + m_1)C_2 + (r_i^2 + m_1 r_i + m_2)C_1}{3r_i^2 + 2m_1 r_i + m_2} e^{r_i \tau}. \tag{9}$$

Here C_1, C_2, C_3 are the values of $\Delta\psi$, its 1st and 2nd derivatives at $\tau = 0$.

It follows from this that by the time $\tau = 1$ the initial deviations change approximately by a factor of $|e^{r_i}|$. Consequently, if: a) all r_i have negative real parts, or b) the real part of r_i is not too large, then the initial deviations will not increase greatly. In the first case it must be

$$m_1 > 0; \qquad m_3 > 0; \qquad m_1 m_2 - m_3 > 0. \tag{10}$$

As for the second case, it should be noted that in our problem it is practically important that during the finite time $\tau = 1$ the initial ···

deviations do not become greater than the specified limiting quantities. If one imposes requirements on the order of growth of the initial deviations over this time, then one can obtain restrictions for the moduli of the coefficients m_i. Using Descartes' rule, we have $\max |r_i| < \max |m_i| + 1$. Thus, if m is the maximum modulus of the coefficients, then the growth over the time τ from 0 to 1 will be, in order of magnitude, no more than e^{m+1} times. From these conditions the parameters S_{10}, S_{11}, S_{12} of the system of automatic control of the projectile can be determined.

The solution of the complete differential equation under zero initial conditions will be

$$\begin{gathered}(\Delta\psi)_{\mathrm{n}} = \frac{1-k^2}{k} \sum_{i=1}^{3} \frac{1}{3r_i^2 + 2m_1 r_i + m_2} \, \frac{1}{\sec\psi_0 - k \operatorname{tg}\psi_0} \times \\ \times \int_{\psi}^{\psi_0} \exp\left\{ r_i \frac{1}{(\operatorname{tg}\psi_0 + \sec\psi_0)^{1/k} (\sec\psi_0 - k\operatorname{tg}\psi_0)} \left[(\operatorname{tg}\omega + \sec\omega)^{1/k} (\sec\omega - k\operatorname{tg}\omega) - \right.\right. \\ \left.\left. - (\operatorname{tg}\psi_0 + \sec\psi_0)^{1/k} (\sec\psi_0 - k\operatorname{tg}\psi_0) \right] \right\} \times \\ \times \left\{ A_1 H + A_2 H \left(\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\omega + \operatorname{tg}\omega} \right)^{1/k} \cos\omega \left(2\sin\omega + \frac{1}{k} \right) + \right. \\ \left. + A_3 H \left(\frac{\sec\psi_0 + \operatorname{tg}\psi_0}{\sec\omega + \operatorname{tg}\omega} \right)^{2/k} \cos^2\omega \left(8\sin^2\omega + \frac{7}{k}\sin\omega + \frac{2-2k^2}{k^2} \right) \right\} d\omega. \end{gathered} \tag{11}$$

The solution of the complete differential equation under zero initial data characterizes the deviations, in the real motion, from the ideal motion, caused by the difference between the actual control law (5) and the ideal one (2).

In flight the angle $\Delta\psi_1$ can be measured, whereas regulation is carried out according to the angle $\Delta\psi$. The dependence between these angles is determined by the formula (Fig. 2)

$$\Delta\psi = \Delta\psi_1 + \operatorname{Arc}\operatorname{tg}\frac{\Delta y\cos\psi - \Delta x\sin\psi}{a + \Delta x\cos\psi + \Delta y\sin\psi}. \tag{12}$$

Knowing $\Delta\psi_1$ from measurements, one can at every instant obtain the value of $\Delta\psi$, since Δx, Δy are found continuously from the last two equations (6), and the angle ψ and the distance to the target a are known from the solution of the ideal motion. At the initial instant of time $\Delta\psi = \Delta\psi_1$, and the values $(\Delta y)_0$ and $(\Delta x)_0$ are equal to zero.

In equation (12) one must not neglect in the denominator the two small terms in comparison with the distance between the projectile and the target a, and, still less, replace the arctangent by the angle, since near the target a tends to zero, and only the exact formula (12) will correctly reflect the process.

The homing system can be self-adjusting (in the terminology of Qian Xuesen ([1]), multistable), if, when the rudder deflection near the target exceeds the permissible value, the projectile speed is reduced so as to satisfy the obtained condition that the rudder turning angle at the target go to zero. Since in this case the projectile speed changes in jumps, provision should also be made for reverse adjustment, increasing the speed if the distance between the target and the projectile begins to increase.

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Leningrad Higher Marine Engineering School
named after Admiral S. O. Makarov

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.