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Abstract

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DEPOSITION OF AEROSOL PARTICLES ON A CYLINDER IN A FLOW UNDER THE ACTION OF ELECTROSTATIC ATTRACTION

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The deposition of aerosol particles on a cylinder under the influence of electric forces plays a role in the operation of high-voltage lines (1) and in the filtration of aerosols by certain fibrous materials (2). For one particular case (see below), this problem was investigated by Koche (1). The deposition of aerosols from a stream on a spherical obstacle under the influence of electrostatic attraction was considered by Koche (3), L. M. Levin (4), and Kramer and Johnston (5).

If we denote by ψ the stream function for the flow of a perpendicular stream around a circular infinite cylinder, by $F(r)$ the force of attraction of a particle to the cylinder, by B the mobility of the particle, and neglect the force of gravity, the inertia of the particle, and its Brownian motion, then the components of the particle velocity in polar coordinates will be

$$v_r = \frac{dr}{dt} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} - BF(r), \qquad v_\theta = \frac{r\,d\theta}{dt} = -\frac{\partial\psi}{\partial r}. \tag{1}$$

From this we obtain the differential equation of the particle trajectory

$$\frac{\partial\psi}{\partial r}\,dr + \frac{\partial\psi}{\partial\theta}\,d\theta = d\psi = BrF(r)\,d\theta, \tag{2}$$

whose integral we shall write in the form

$$\psi = f(\theta, C) \tag{3}$$

(C is the integration constant).

As $\theta \to \pi$, the particle trajectory follows the streamline characterized by the value $\psi = f(\theta = \pi, C)$. The coefficient ε of capture of particles by the cylinder is equal to

$$\varepsilon = \frac{\psi_{\text{gr}}}{v_0 a} = \frac{f(\theta = \pi, C_{\text{gr}})}{v_0 a}, \tag{4}$$

where C_{gr} is the value of C for the limiting trajectory of particles still captured by the cylinder, v_0 is the velocity of the undisturbed stream, and a is the radius of the cylinder.

Let $r_0 = a + R$, where R is the particle radius. A particle moving along the limiting trajectory must, at the point of capture $(r = r_0, \theta = \theta_0)$, have $v_r \leq 0$. For $v_r < 0$ the limiting trajectory intersects, and for $v_r = 0$ it touches, the surface $r = r_0$. If, at $r = r_0$, the condition $v_r < 0$ is satisfied for any value of θ from 0 to π, then the limiting trajectory passes through the point with coordinates $r = r_0$ and $\theta = \theta_0 = 0$. Substituting these values into (3) and taking into account that for $\theta = 0$ and $r = r_0$, $\psi = 0$, we obtain the expression for determining C_{gr}

$$f(\theta = 0, C_{\text{gr}}) = 0. \tag{5}$$

which is valid when $v_r \leq 0$ at $r = r_0$ and $\theta = 0$, i.e. when

$$\frac{B r_0 F(r_0)}{(\partial\psi/\partial\theta)_{r_0, \theta=0}} \geq 1. \tag{6}$$

If at $r = r_0$ the condition $v_r \leq 0$ is satisfied only for $\theta \geq \theta_0$, then the limiting trajectory passes through the point with coordinates $r = r_0$ and $\theta = \theta_0$, where the value of θ_0 is determined by the equality $v_r = 0$ at $r = r_0$ and $\theta = \theta_0$. Substituting these coordinate values into (3), we obtain an expression for determining C_{gr}:

$$\psi(r_0, \theta_0) = f(\theta_0, C_{\text{gr}}), \tag{7}$$

which is valid when $v_r \geq 0$ at $r = r_0$ and $\theta = 0$, and in which θ_0 is determined, according to (1), by the expression

$$\left(\frac{\partial\psi}{\partial\theta}\right)_{r_0, \theta_0} = B r_0 F(r_0). \tag{8}$$

Fig. 1

Fig. 1

Figure 1: Fig. 1

The values of the capture coefficient ε are calculated from the found values of C_{gr} by means of (4). The formulas obtained are applicable for any type of central forces between the cylinder and the particles.

Let us now consider several specific problems.

1. **Charged cylinder and charged particles.** If ρ denotes the volume density of charge of the cylinder, and e the charge of a particle, then, neglecting induction forces, $F(r) = 2\pi\rho e a^2/r = \alpha/r$. Integration of (2) in this case gives

$$\psi = f(\theta, C) = B\alpha\theta + C. \tag{9}$$

For parameter relationships of the problem under which condition (6) is satisfied, the value of C_{gr}, according to (5) and (9), is $C_{\text{gr}} = 0$, while the value of ε, according to (4), is

$$\varepsilon = \frac{B\alpha\pi}{v_0 a} = \frac{\pi a\rho e}{3v_0\eta R}. \tag{10}$$

Here B is determined by Stokes' drag law, and η is the viscosity of the medium. The value of ε obtained does not depend on the form of the function ψ.

The form of condition (6) and the expression for ε when (6) is not satisfied, on the contrary, depend on the character of the flow. If (6) is not satisfied, then, according to (4), (9), and (7),

$$\varepsilon = \frac{B\alpha(\pi - \theta_0) + \psi(r_0, \theta_0)}{v_0 a}.$$

For potential flow ($\psi = v_0(r - a^2/r)\sin\theta$), the value of θ_0, according to (8), is determined by the expression

$$B\alpha = v_0(r_0 - a^2/r_0)\cos\theta_0,$$

and condition (6) has the form

$$B\alpha \geq v_0(r_0 - a^2/r_0).$$

Figure 1 gives particle trajectories for potential flow, calculated according to formula (9).

2. **Charged cylinder and uncharged particles.** If the effect of the polarized particle on the field of the cylinder is neglected, then

$$F(r) = 4\pi^2\rho^2a^4\frac{D_1-1}{D_1+2}\left(\frac{R}{r}\right)^3 = \frac{\beta}{r^3},$$

where D_1 is the dielectric constant of the particle. Equation (2) takes the form

$$d\psi = \frac{B\beta}{r^2}\,d\theta. \tag{11}$$

For $r/a \gg 1$ ($\varepsilon \gg 1$) the cylinder does not affect the flow, $\psi = v_0 r\sin\theta$, and (11) is transformed into

$$\psi^2 d\psi = B\beta v_0^2 \sin^2\theta\,d\theta,$$

whence

$$\psi = f(\theta, C) = \left[\frac{3}{2}B\beta v_0^2\left(\theta - \frac{1}{2}\sin 2\theta\right) + C\right]^{1/3}.$$

Condition (6) takes the form $B\beta \geq v_0 r_0^3$. When it is met,

when it is satisfied, the value of ε according to (5) and (4) is equal to

$$\varepsilon = \left(\frac{3B\beta\pi}{2v_0a^3}\right)^{1/3} = \left(\frac{D_1-1}{D_1+2}\frac{\pi^2\beta^2aR^2}{v_0\eta}\right)^{1/3}. \tag{12}$$

If (6) is not satisfied, the value of ε can be determined from (4) and (7), substituting into the latter the value $\cos\theta_0 = B\beta/v_0r_0^3$, obtained from (8). The solution for $r/a \gg 1$ coincides with that obtained by Koshe [(1)] for capture by a charged cylinder of falling drops, if instead of v_0 one substitutes the Stokes settling velocity of the drops.

For $(r-a)/a = x \ll 1$ ($\varepsilon \ll 1$), $r \sim a$, and equation (11) takes the form $a^2 d\psi = B\beta\,d\theta$. Integration gives $\psi = f(\theta, C) = B\beta\theta/a^2 + C$. When condition (6) is satisfied, the value of C_{rp} according to (5) is $C_{\text{rp}} = 0$. The capture coefficient is equal to

$$\varepsilon = \frac{B\beta\pi}{v_0a^3} = \frac{D_1-1}{D_1+2}\frac{2\pi^2\beta^2aR^2}{3v_0\eta}. \tag{13}$$

This expression does not depend on the character of the function ψ. The expressions for condition (6) and for ε when (6) is not satisfied, on the contrary, depend on the character of the flow. Condition (6) for potential flow ($\psi = 2v_0 ax \sin\theta$) takes the form $B\beta \geqslant 2v_0 a^3 x_0$, where $x_0 = (r_0 - a)/a = R/a$, and for viscous flow ($\psi = 2wax^2 \sin\theta$, where $2w = v_0/(2.00 - \ln \mathrm{Re})$ and $\mathrm{Re} = 2v_0 a/\nu$), $B\beta \geqslant 2wa^3 x_0^2$. If (6) is not satisfied, according to (4) and (7),

$$\varepsilon = \left[\frac{B\beta}{a^2}(\pi - \theta_0) + \psi(r_0, \theta_0)\right] / v_0 a,$$

where for potential flow $\cos\theta_0 = B\beta/2v_0 a^3 x_0$, and for viscous flow $\cos\theta_0 = B\beta/2wa^3 x_0^2$.

3. **Uncharged cylinder and charged particles.** For $x \ll 1$ and $r - a \gg R$ ($\varepsilon \ll 1$), the interaction of a particle and a cylinder is equivalent to the interaction of a point charge and a plane. In this case

$$F(r) = \frac{D_2 - 1}{D_2 + 1} \frac{e^2}{4ax^2} = \frac{\gamma}{x^2},$$

where D_2 is the dielectric constant of the cylinder. Equation (2) takes the form

$$d\psi = \frac{B\gamma a}{x^2}\, d\theta. \tag{14}$$

For potential flow, after eliminating x, equation (14) is transformed into

$$\psi^2 d\psi = 4B\gamma v_0^2 a^3 \sin^2\theta\, d\theta,$$

which gives

$$\psi = f(\theta, C) = \left[6B\gamma v_0^2 a^3 \left(\theta - \frac{1}{2}\sin 2\theta\right) + C\right]^{1/3}.$$

Condition (6) takes the form $B\gamma \geqslant 2v_0 x_0^3$. When it is satisfied, the value of ε according to (4) and (5) is equal to

$$\varepsilon = \left(\frac{6B\gamma\pi}{v_0}\right)^{1/3} = \left(\frac{D_2 - 1}{D_2 + 1} \frac{e^2}{4v_0 a^2 \eta R}\right)^{1/3}. \tag{15}$$

If (6) is not satisfied, the expression for ε is obtained from (4) and (7) by substituting into the latter the value $\cos\theta_0 = B\gamma/2v_0 x_0^3$, obtained from (8).

For viscous flow, after eliminating x, equation (14) is transformed into $\psi\, d\psi = 2B\gamma wa^2 \sin\theta\, d\theta$, whence $\psi = f(\theta, C) = (-4B\gamma wa^2 \cos\theta + C)^{1/2}$. Condition (6) has the form $B\gamma \geqslant 2wx_0^4$. When it is satisfied, according to (5) $C_{\mathrm{rp}} = 4B\gamma wa^2$, which gives, with the aid of (4),

$$\varepsilon = \left(\frac{8B\gamma w}{v_0^2}\right)^{1/2} = \left(\frac{D_2-1}{D_2+1}\frac{e^2 w}{3\pi v_0^2 a^2 \eta R}\right)^{1/2}. \tag{16}$$

If (6) is not satisfied, taking into account that according to (8) $\cos\theta_0 = B\gamma/2wx_0^4$, we obtain from (7) $C_{\text{rp}} = 4w^2a^2x_0^4 + B^2\gamma^2a^2x_0^{-4}$. Substituting into (4), we obtain

$$\varepsilon = 2\frac{w}{v_0}x_0^2 + \frac{B\gamma}{v_0 x_0^2} = 2\frac{w}{v_0}\left(\frac{R}{a}\right)^2 + \frac{D_2-1}{D_2+1}\frac{e^2}{24\pi v_0 \eta R^3}. \tag{17}$$

The first term of this expression corresponds to capture due to the purely interception effect.

4. Van der Waals interaction between the cylinder and the particle. This effect may play a role in the filtration of aerosols by fibrous materials ([6]). According to ([6]), the force of van der Waals interaction between a particle and a plane ($\varkappa \ll 1$, $\varepsilon \ll 1$) is equal to $F = 2\pi^2ARh^2/3(h^2-R^2)^2$, where A is the interaction constant (of order 10^{-14}–10^{-13} erg), and h is the distance from the center of the particle to the plane. However, this expression contains an error that crept into the differentiation in the transition from formula (5) to formula (6) of ([6]), and the corrected value is $F = 2\pi^2AR^3/3(h^2-R^2)^2$. In the approximation $a \gg r-a \gg R$, $F(r) = 2\pi^2AR^3/3a^4\varkappa^4 = k/\varkappa^4$. Equation (2) takes the form

$$d\psi = \frac{Bka}{\varkappa^4}\,d\theta. \tag{18}$$

For potential flow, eliminating $\varkappa$ gives $\psi^4 d\psi = 16Bkv_0^4a^5\sin^4\theta\,d\theta$, whence

$$\psi = f(\theta, C) = \left[10Bkv_0^4a^5\left(3\theta - 2\sin 2\theta + \frac{1}{4}\sin 4\theta\right) + C\right]^{1/5}.$$

Condition (6) gives $Bk \gg 2v_0\varkappa_0^5$.

When it is satisfied, according to (4) and (5), ε is equal to

$$\varepsilon = \left(\frac{30Bk\pi}{v_0}\right)^{1/5} = \left(\frac{10\pi^2AR^2}{3v_0a^4\eta}\right)^{1/5}. \tag{19}$$

When (6) is not satisfied, the expression for ε is obtained from (4) and (7) by substituting $\cos\theta_0 = Bk/2v_0\varkappa_0^5$.

For viscous flow, eliminating $\varkappa$ from (18) gives $\psi^2 d\psi = 4Bka\omega^2a^3\sin^2\theta\,d\theta$. On integration we obtain

$$\psi = f(\theta, C) = \left[6Bka\omega^2a^3\left(\theta - \frac{1}{2}\sin 2\theta\right) + C\right]^{1/3}.$$

Condition (6) gives $Bk \gg 2\omega\varkappa_0^6$. When this condition is satisfied, according to (4) and (5), ε is equal to

$$\varepsilon = \left(\frac{6Bka\omega^2\pi}{v_0^3}\right)^{1/3} = \left(\frac{2\pi^2\omega^2 AR^2}{3v_0^3 a^4\eta}\right)^{1/3}. \tag{20}$$

When (6) is not satisfied, the expression for ε is obtained from (4) and (7) by substituting the value $\cos\theta_0 = Bk/2\omega\varkappa_0^6$, obtained from (8).

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Note: Figure translations are in progress. See original paper for figures.