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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

Full Text

Physics

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Dispersion Relations for Virtual Processes

(Presented by Academician N. N. Bogoliubov, 18 VII 1957)

1. One of the tasks of quantum field theory is to establish relations between observable quantities. Therefore much attention is being devoted to the so-called dispersion relations, which are consequences of the general principles of local field theory and are not connected with any particular form of perturbation theory.

At the present time dispersion relations have been obtained for processes of meson-nucleon scattering, photoproduction, the Compton effect on a nucleon, etc. ⁽¹⁾. It seems very promising to establish dispersion relations for vertex parts—“blocks”—entering into the matrix elements of processes. The simplest examples may be the “blocks” of photoproduction and of the Compton effect. Schematically they are shown in Fig. 1.

The photoproduction “block” has one virtual γ -quantum, while the Compton-effect block may have either one virtual quantum (the second quantum is assumed to be real) or two virtual quanta. The “blocks” shown in Fig. 1 differ from the real processes of photoproduction and the Compton effect in that in this case $k^2 \neq 0$. Obviously, they are component parts of the matrix elements of real physical processes. Thus, for example, the vertex part of photoproduction may enter into the processes schematically shown in Fig. 2, and that of the Compton effect into the processes shown in Fig. 3. Many other processes can be cited in which the photoproduction and Compton-effect “blocks” will enter as component parts. The consideration of block elements is to a considerable degree analogous, and therefore we shall restrict ourselves to considering only the photoproduction “block” with $k^2 \neq 0$.

Fig. 1

Fig. 2

Fig. 3

Figure 3: Fig. 3

Fig. 3

For the photoproduction “block” dispersion relations can be obtained which, as in the case of real photoproduction, may be used to derive approximate equations. Since the “block” is a component part of the matrix elements of certain processes, these equations may provide information about those processes.

2. According to the general theory of dispersion relations, in the coordinate system ($\mathbf{p} + \mathbf{p}' = 0$) the “Hermitian part” of the amplitude is equal to a certain integral over the energy of the “anti-Hermitian part” plus an arbitrary polynomial $P_n(E)$.

As in the case of real photoproduction, the “anti-Hermitian” part of the amplitude for photoproduction of π -mesons by virtual γ -quanta (for example, for $k^2 \leq 0$) can be written in the form

$$\begin{aligned}
 A_{\nu\rho}(E, \lambda\mathbf{e}) = & \pi \sum_n \langle p', s' | j_\rho(0) | n, \lambda\mathbf{e} - \varepsilon\mathbf{p} \rangle \langle n, \lambda\mathbf{e} - \varepsilon\mathbf{p} | j_\nu(0) | p, s \rangle \times \\
 & \times \delta(\sqrt{M^2 + \mathbf{p}^2} + E - \sqrt{M_n^2 + \lambda^2 + \varepsilon^2 \mathbf{p}^2}) - \\
 & - \pi \sum_n \langle p', s' | j_\nu(0) | n, -\lambda\mathbf{e} + \varepsilon\mathbf{p} \rangle \langle n, -\lambda\mathbf{e} + \varepsilon\mathbf{p} | j_\rho(0) | p, s \rangle \times \\
 & \times \delta(\sqrt{M^2 + \mathbf{p}^2} - E - \sqrt{M_n^2 + \lambda^2 + \varepsilon^2 \mathbf{p}^2}), \quad (1)
 \end{aligned}$$

where $j_\rho(0)$, $j_\nu(0)$ are the “meson” and “electromagnetic” currents, E is the energy of the π -meson,

$$\varepsilon = \frac{1}{4} \cdot \frac{m_\pi^2 + m_\gamma^2}{\mathbf{p}^2}; \quad \lambda^2 = E^2 - (1 - \varepsilon)^2 \mathbf{p}^2 - m_\pi^2; \quad k^2 = (q - q')^2 = -m_\gamma^2.$$

The region of negative energies in the dispersion relations can be reduced to the region of positive energies if one uses the parity properties with respect to energy of the photoproduction “block.” We note that the parity properties of the photoproduction “block” are analogous to the parity properties of the real-photoproduction amplitude.

If one makes the usual assumption that between M and $M + m_\pi$ there are no bound states of the meson-nucleon system, then it is easy to show that the integration region in the dispersion relations splits into two parts

$$0 < E' < \frac{Mm_\pi + \frac{1}{4}(m_\pi^2 + m_\gamma^2) - \mathbf{p}^2}{\sqrt{M^2 + \mathbf{p}^2}} \leq E' < \infty. \quad (2)$$

In the first region, only one-nucleon states ($n = 0$) give a nonzero contribution to the integral. All states with $n \geq 1$ contribute only to the integral over the second region. For scatterer momenta $\mathbf{p}^2 < \frac{1}{4}(m_\gamma^2 - m_\pi^2)$, the anti-Hermitian part of the “block” in the first region will be

$$A_{\nu\rho}(E, \lambda\mathbf{e}) = \pi \sum_{s''} \langle p' s' | j_\rho(0) | \lambda\mathbf{e} - \varepsilon\mathbf{p} \rangle \langle \lambda\mathbf{e} - \varepsilon\mathbf{p}, s'' | j_\nu(0) | p, s \rangle \times \\ \times \frac{M^2 - \frac{1}{4}(m_\pi^2 - m_\gamma^2)}{M^2 + \mathbf{p}^2} \delta\left(E + \frac{\mathbf{p}^2 + \frac{1}{4}(m_\pi^2 - m_\gamma^2)}{\sqrt{M^2 + \mathbf{p}^2}}\right), \quad (3)$$

and for momenta

$$\frac{1}{4}(m_\gamma^2 - m_\pi^2) < \mathbf{p}^2 < \frac{1}{2}Mm_\pi + \frac{1}{4}m_\gamma^2$$

$$A_{\nu\rho}(E, \lambda\mathbf{e}) = -\pi \sum_{s''} \langle p', s' | j_\nu(0) | -\lambda\mathbf{e} + \varepsilon\mathbf{p}, s'' \rangle \langle -\lambda\mathbf{e} + \varepsilon\mathbf{p}, s'' | j_\rho(0) | p, s \rangle \times \\ \times \frac{M^2 - \frac{1}{4}(m_\pi^2 - m_\gamma^2)}{M^2 + \mathbf{p}^2} \delta\left(E - \frac{\mathbf{p}^2 + \frac{1}{4}(m_\pi^2 - m_\gamma^2)}{\sqrt{M^2 + \mathbf{p}^2}}\right). \quad (4)$$

From considerations of relativistic invariance, the meson and electromagnetic currents can be written in the form

$$\langle p' s' | j_\rho(0) | ps \rangle = g \langle \bar{u}_{s'}(p') \gamma_5 \tau_\rho u_s(p) \rangle;$$

g is the renormalized pseudoscalar coupling constant;

$$\langle p', s' | j_\nu(0) | p, s \rangle = \langle \bar{u}_{s'}(p') \left\{ e \frac{1 + \tau_3}{2} \gamma_\nu F_1(k^2) + \right. \\ \left. + \frac{1}{2} \left(\frac{1 + \tau_3}{2} F_2^p(k^2) + \frac{1 - \tau_3}{2} F_2^n(k^2) \right) [(k\gamma), \gamma^\nu] \right\} u_s(p) \rangle.$$

It should be emphasized that the invariant functions $F_1(k^2)$, $F_2^{p,n}(k^2)$ are form factors characterizing the electromagnetic structure of the nucleon. Information about these functions can be obtained from the process of electron scattering on a nucleon ⁽²⁾. It is easy to see that the dispersion relations obtained still contain

a region of unobservable energies. In the case where the recoil momentum of the nucleon is

$$\mathbf{p}^2 = \frac{1}{4} (m_\pi^2 + m_\gamma^2) \frac{M}{M + m_\pi},$$

the unobservable region, as in the case of real photoproduction, is completely absent.

One of the important applications of dispersion relations is the derivation of an approximate system of equations for certain processes. For meson-nucleon scattering and photoproduction, such equations have been obtained both in the approximation of a fixed nucleon source and with allowance for nucleon recoil⁽³⁾.

Analogous equations can also be obtained for the block elements, with the only difference that instead of the unitarity condition one must use expression (1). It is easy to see that expression (1) (as well as the unitarity condition for photoproduction) makes it possible, in the one-meson approximation, to express the phases of the photoproduction "block" through the phases of the meson-nucleon scattering process.

Let us note that especially simple equations of the Low type can be obtained in the approximation of a fixed nucleon source, since in this case the unobservable energy region is completely absent.

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CITED LITERATURE

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