Electrical Engineering

A. D. KHARKEVICH

1957

SovietRxiv

Abstract

Full Text

Electrical Engineering

A. D. KHARKEVICH

MULTISTAGE CONSTRUCTION OF FULLY ACCESSIBLE SWITCHING SYSTEMS

(Presented by Academician V. S. Kulebakin, 22 VI 1956)

1. This article gives a general method for constructing fully accessible switching systems and presents recurrence relations that make it possible to estimate a switching system by the number of crosspoints.

2. By a switching system we shall mean a discrete-action system having M inputs and N outputs, which at required moments in time can create connecting paths between inputs and outputs. Such a switching system may be, for example, a contact multi-terminal network, studied by the methods of the theory of relay-contact circuits ([1,2]). In technical applications, ordered switching systems are most often encountered, in which there is a definite regularity in the construction of connecting paths between inputs and outputs. Thus, in automatic telephony, switching systems with a symmetric construction of connecting paths with respect to each of the inputs and outputs are widely used.

We shall call a switching system fully accessible if it provides the possibility of creating a connecting path between any input and any output, independently of the number of paths established by the moment under consideration. In this case the maximum possible number of paths is limited by the smaller of the numbers M and N.

3. Taking into account the technical realization of a switching system, one may regard as one of the indicators of the efficiency of its construction the total number of crosspoints T. The trivial construction of a fully accessible switching system in the form of a switch*, for which the total number of crosspoints T_1 is equal to the product of the number of inputs M by the number of outputs N ($T_1 = MN$), is not always the most efficient.

Clos ([3]) proposed an efficient method for constructing fully accessible switching systems which, however, can be represented as a special case of a construction of a more general form (multistage construction).

4. A multistage construction of a switching system can be obtained by repeated use of the construction rule proposed in ([3]).

The switching system shown in Fig. 1a and 1b, consisting of separate switches (shown in Fig. 1b as rectangles), is transformed so that each of the switches of this system is replaced by a fully accessible switching subsystem constructed according to the same rule as the entire system. Such a transformation is shown in Fig. 2, where each of the rectangles denotes the switching system of Fig. 1. The system of Fig. 2, obtained by twofold application of one and the same construction rule, will be called two-stage. If such a process is repeated s times,

* By a switch is understood a fully accessible switching system in which the connecting path between an input and an output is closed at a single crosspoint.

then we obtain an s-stage construction of a fully accessible switching system.

5. In a two-stage construction, the total number of connection points is a function of the capacity (M and N) of the switching system, of the parameter n of the system, and of the parameters $n_{1,1}$ and $n_{1,2}$ of the subsystems of which the system is composed, and is expressed as follows:

Fig. 1

$$T_2(M;\,N;\,n) = \frac{M+N}{n}\,T(n;\,2n-1;\,n_{1,1}) + (2n-1)\,T\left(\frac{M}{n};\,\frac{N}{n};\,n_{1,2}\right), \quad (1)$$

where M is the number of inputs, and N is the number of outputs of the switching system under consideration, for which a two-stage construction is used; n is the parameter of the system (the number of inputs to any subsystem of the first stage of the system, or the number of outputs from any subsystem of the third stage); $T(n;\,2n-1;\,n_{1,1})$ is the total number of connection points of any subsystem (with a single-stage construction) of the first and third stages of the system ($n_{1,1}$ is the parameter of the subsystem); $T\left(\frac{M}{n};\,\frac{N}{n};\,n_{1,2}\right)$ is the total number of connection points of any subsystem (with a single-stage construction) of the second stage of the system ($n_{1,2}$ is the parameter of the subsystem).

If it is taken into account that, for the total number of connection points in a single-stage construction ([3]), the relation

$$T_1(X;\,Y;\,\alpha) = T(X;\,Y;\,\alpha) = (2\alpha-1)\left(X+Y+\frac{XY}{\alpha^2}\right), \quad (2)$$

holds, where X is the number of inputs, Y is the number of outputs, and α is the parameter of a single-stage switching system, then relation (1), taking (2) into account, can be

written in the following form:

Fig. 2

Figure 1: Fig. 2

$$T_{3^2}(M;N;n) = \frac{M+N}{n}(2n_{1,1}-1)\left[3n-1+\frac{n(2n-1)}{n_{1,1}^2}\right]+ \\ +(2n-1)(2n_{1,2}-1)\left(\frac{M+N}{n}+\frac{MN}{n^2n_{1,2}^2}\right). \quad (3)$$

For the special case of a rectangular system ($M = N$), (3) takes the form:

$$T_{3^2}(N;N;n) = 2\frac{N}{n}(2n_{1,1}-1)\left[3n-1+\frac{n(2n-1)}{n_{1,1}^2}\right]+ \\ +(2n-1)(2n_{1,2}-1)\left(2\frac{N}{n}+\frac{N^2}{n^2n_{1,2}^2}\right). \quad (4)$$

Fig. 2

6. In the general case, for an s-stage construction, the total number of connection points $T_3^s(M;N;n)$—a function of the capacity of the system (M and N) and of the 2^s-1 parameters n; $n_{1,1}, n_{1,2}$; $n_{2,1}, n_{2,2}, n_{2,3}, n_{2,4}; \dots$; $n_{s-1,1}, n_{s-1,2}, \dots,$ $n_{s-1,2^{s-1}}$—can be obtained from the relation

$$T_3^s(M;N;n) = \frac{M+N}{n}T_{3^{s-1},1}(n;2n-1;n_{1,1})+$$

$$+(2n-1)T_{3^{s-1},2}\left(\frac{M}{n};\frac{N}{n};n_{1,2}\right), \quad (5)$$

where

$$T_{3^{s-1},i}(M_{1,i};N_{1,i};n_{1,i}) =$$

$$=\frac{M_{1,i}+N_{1,i}}{n_{1,i}}T_{3^{s-2}}(n_{1,i};2n_{1,i}-1;n_{2,2i-1})+$$

$$+(2n_{1,i}-1)T_{3^{s-2}}\left(\frac{M_{1,i}}{n_{1,i}};\frac{N_{1,i}}{n_{1,i}};n_{2,2i}\right) \qquad (i=1,2)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$T_{3,i}(M_{s-1,i}; N_{s-1,i}; n_{s-1,i}) =$$

$$= \frac{M_{s-1,i} + N_{s-1,i}}{n_{s-1,i}} T_{1,i}(n_{s-1,i}; 2n_{s-1,i} - 1) +$$

$$+ (2n_{s-1,i} - 1) T_{1,i} \left(\frac{M_{s-1,i}}{n_{s-1,i}}; \frac{N_{s-1,i}}{n_{s-1,i}} \right)$$

$$(i = 1, 2, \ldots, 2^{s-1}).$$

7. Applying the principle of multistage construction only to part of the switches of the system, we obtain a partial multistage construction. Thus, under repeated transformation of the intermediate switches in each of the switching systems, we obtain one of the special cases of partial multistage construction, for which the total number of crosspoints is a function of the capacity of the switching system (M and N) and of $\frac{t-1}{2}$ construction parameters, and is expressed as follows:

$$T_t \left(M; N; n; n_1; \ldots; n_{\frac{t-3}{2}} \right) =$$

$$= (2n-1)(M+N) + (2n-1)(2n_1-1)\frac{M+N}{n} + \cdots$$

$$\cdots + (2n-1)(2n_1-1)\cdots(2n_{\frac{t-5}{2}}-1)\frac{M+N}{nn_1 \cdots n_{\frac{t-7}{2}}} +$$

$$+ (2n-1)(2n_1-1)\cdots(2n_{\frac{t-3}{2}}-1)\left(\frac{M+N}{nn_1 \cdots n_{\frac{t-5}{2}}} + \frac{MN}{n^2 n_1^2 \cdots n_{\frac{t-3}{2}}^2} \right), \qquad (6)$$

where t is the number of crosspoints in each connecting path between an input and an output of the system.

In the special case $M = N$ and when the condition

$$n = n_1 = n_2 = \cdots = n_{\frac{t-3}{2}} = N^{\frac{2}{t+1}} \qquad (7)$$

is satisfied, formula (6) takes the form

$$T_t = 2\sum_{k=2}^{\frac{t+1}{2}} N^{\frac{2k}{t+1}} \left(2N^{\frac{2}{t+1}} - 1\right)^{\frac{t+3}{2}-k} + N^{\frac{4}{t+1}} \left(2N^{\frac{2}{t+1}} - 1\right)^{\frac{t-1}{2}},$$

obtained by Clos [3].

Laboratory for the Development
of Scientific Problems of Wire Communication
Academy of Sciences of the USSR

Received
22 VI 1956

REFERENCES

1. M. A. Gavrilov, *Theory of Relay-Contact Circuits*, Moscow-Leningrad, 1950.

2. E. N. Gilbert, BSTJ, **30**, No. 3, 668 (1951).

3. C. Clos, BSTJ, **32**, No. 2, 406 (1953).

Note: Figure translations are in progress. See original paper for figures.