On the Forecasting of Atmospheric Pressure with the Aid of High-Speed Electronic Computing Machines

1957

SovietRxiv

Abstract

Full Text

Geophysics

S. A. Mashkovich

On the Forecasting of Atmospheric Pressure with the Aid of High-Speed Electronic Computing Machines

(Presented by Academician A. A. Dorodnitsyn, 9 VIII 1956)

The studies of I. A. Kibel, N. I. Buleev, G. I. Marchuk, and others made it possible to formulate the problem of short-range forecasting of meteorological elements. However, before the advent of high-speed computing machines it was impossible to apply rational methods for solving this problem. The present communication sets forth a forecasting method based on the use of modern computational facilities.

The problem of short-range forecasting of meteorological elements over a "flat" Earth can be reduced to the solution of the following system of equations:

$$\frac{\partial \Omega}{\partial t} + u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} + \beta v = \frac{l}{P}\frac{\partial \tau}{\partial \zeta}; \tag{1a}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\alpha \overline{T}}{P\zeta}\tau; \quad (1\ )$$

$$u = -\frac{1}{l}\frac{\partial H}{\partial y}, \qquad v = \frac{1}{l}\frac{\partial H}{\partial x}; \quad (1\ )$$

$$T = -\frac{\zeta}{R}\frac{\partial H}{\partial \zeta}. \quad (1\ )$$

Here and below the following notation is adopted: x, y are horizontal coordinates; p is pressure; $\zeta = p/P$ is the vertical coordinate; $P = 1000$ mb; t is time; H is geopotential; T is temperature; u, v are the components of wind velocity along the axes x, y; R is the gas constant; $\alpha = R(\gamma_a - \gamma)/g$, γ_a is the dry-adiabatic gradient; γ is the vertical temperature gradient; g is the acceleration due to gravity; l is the Coriolis parameter; w is the vertical component of velocity; $\overline{T}$ is the mean value of the temperature; $c^2 = \alpha R\overline{T}$;

$$\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}; \qquad \beta = \frac{dl}{dy}; \qquad \tau = \frac{gp}{R\overline{T}}\left(\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y} - w\right);$$

Fig. 1a

Figure 1: Fig. 1a

the symbol (a, b) denotes

$$\frac{\partial a}{\partial x}\frac{\partial b}{\partial y} - \frac{\partial a}{\partial y}\frac{\partial b}{\partial x}; \qquad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Equation (1a) describes changes in the vertical component of the vortex of velocity in a given air particle; equation (1) means that an adiabatic process is being considered. Equations (1) represent the formula for the geostrophic wind. These relations are obtained as a result of simplifying the equations of motion. The closeness of the wind to the geostrophic wind is well known to meteorologists.

The static equation (1) makes it possible to eliminate the temperature from (1). Replacing, further, in (1a) and (1) u and v by formulas (1) and then eliminating τ from (1a), we obtain the following basic equation for determining H:

$$\frac{\partial}{\partial \zeta}\left(\zeta^2 \frac{\partial^2 H}{\partial \zeta\, \partial t}\right) + \frac{c^2}{l^2}\frac{\partial \Delta H}{\partial t} = -\frac{c^2}{l^2}\left[\frac{1}{l}(H, \Delta H) + \beta \frac{\partial H}{\partial x}\right] - \frac{1}{l}\frac{\partial}{\partial \zeta}\left[\zeta^2\left(H, \frac{\partial H}{\partial \zeta}\right)\right]. \tag{2}$$

The boundary conditions adopted are:

1) The vertical velocity is equal to zero at the earth' s surface, which is approximately written in the form: for $\zeta = 1$

$$\frac{\partial^2 H}{\partial \zeta\, \partial t} + \alpha \frac{\partial H}{\partial t} = \frac{1}{l}\left(\frac{\partial H}{\partial \zeta}, H\right).$$

2) At the upper boundary of the atmosphere the mass flux in the vertical direction is equal to zero; this condition reduces to the requirement that the expression

$$\zeta \frac{\partial^2 H}{\partial t\, \partial \zeta}$$

be bounded as $\zeta \to 0$.

3) A solution is sought that is bounded as $\sqrt{x^2 + y^2} \to \infty$. It is obvious that at the initial moment (for $t = t_0$) the geopotential H must be specified throughout the entire space.

Fig. 1b

Figure 2: Fig. 1b

Fig. 1c

Figure 3: Fig. 1c

Fig. 1a

The solution of equation (2) is sought in the following way. Suppose that in (2) the desired function is $\partial H/\partial t$, while the derivatives of H with respect to the coordinates are known. Then the problem of finding $\partial H/\partial t$ reduces to solving an equation of elliptic type with the boundary conditions indicated above. The solution is written in the form

$$\frac{\partial H}{\partial t} = \int_0^1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[(H, \Delta H) + \beta l \frac{\partial H}{\partial x} \right] M_1(x, y, \zeta, x', y', \zeta') + \left(H, \frac{\partial H}{\partial \zeta} \right) M_2(x, y, \zeta, x', y', \zeta') \right\} dx'\, dy'\, d\zeta'. \tag{3}$$

Here M_1 and M_2 are Green' s functions, expressible in terms of elementary transcendental functions. The analytic expression for M_1 and M_2 was first obtained by N. I. Buleev and G. I. Marchuk several years ago.

Formula (3) gives the relation between $\partial H/\partial t$ and the derivatives of H with respect to the coordinates for any moment of time. According to the initial condition indicated above, for $t = t_0$ the geopotential is specified throughout the entire space. Consequently, with the aid of (3) one can find the field

$$\left.\frac{\partial H}{\partial t}\right|_{t=t_0}.$$

Using further the formula

Fig. 1b

Fig. 1c

$$H|_{t=t_0+\delta t} = H|_{t=t_0} + \left.\frac{\partial H}{\partial t}\right|_{t=t_0} \delta t,$$

it is possible to determine the field H for the time $t+\delta t$. Repeating the described sequence of operations several times, one can find the values of H for the time of interest to us.

The method of solving equation (2) set forth above was used as the basis of the forecasting scheme we developed. Solving the problem by this method is possible only with the use of a high-speed computer. We carried out the computations on the BESM machine of the Academy of Sciences of the USSR.

The use in the computations of a large number of values of H proves practically inconvenient. Therefore two additional simplifications were introduced. First, a model of a polytropic atmosphere was used. In this case the geopotential height of any isobaric surface can be found from the heights of two arbitrarily chosen surfaces. Consequently, in computations by (3) it is sufficient to use data only at two levels in the atmosphere. Second, initial data were used only for the territory of Europe and Western Siberia. In accordance with the above, the fields of surface pressure and AT 500, specified at 24×19 points, were taken as initial data; the distance between points was 250 km. The derivatives with respect to the horizontal coordinates in (3) were replaced by finite differences, and the integration with respect to ζ' in (3) was carried out analytically.

In the computations it was assumed that the geopotential does not change in two rows of points bordering the field. This condition was introduced because, when initial data are used only over part of the Northern Hemisphere, it is impossible to compute a forecast for the edges of the region under consideration.

An essential question is the determination of the conditions for stability of the solution and the choice of the time step δt. As a result of experimental computations for the given model, the step $\delta t = 22.5$ min was chosen. In computations with a larger step (for example, $\delta t = 45$ min) the solution proves unstable and parasitic waves arise. Experience also shows that, after every 10–15 "time steps" have been carried out, it is useful to perform a comparatively weak smoothing of the field H, in order to exclude small disturbances from consideration.

As an example, we give a forecast from the initial data for the evening of 13 X 1954. The surface-pressure chart for 18 h on 13 X is shown in Fig. 1a, and the chart for 18 h on 14 X in Fig. 1b. Figure 1c presents the forecast of surface pressure for 18 h on 14 X, computed on BESM. The computations required to perform one "time step" took 35 sec of BESM operation. The total time for computing a forecast for one day, including data input and printing of the results, was about 40 min; during this period about 20,000,000 operations were performed.

A characteristic feature of the change in the baric field over the day considered was the rapid displacement of a cyclone, located at the initial time over the North Sea, into the region of Finland, and its merger with the extensive depression situated there. This feature of the process was correctly obtained in the forecast.

In conclusion I consider it my pleasant duty to thank S. L. Belousov for the assistance rendered in carrying out this work.

Received
8 VI 1956

Note: Figure translations are in progress. See original paper for figures.