ON THE INFLUENCE OF INITIAL IMPERFECTIONS ON THE STABILITY OF CYLINDRICAL SHELLS UNDER EXTERNAL PRESSURE

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Abstract

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A. S. VOL' MIR

ON THE INFLUENCE OF INITIAL IMPERFECTIONS ON THE STABILITY OF CYLINDRICAL SHELLS UNDER EXTERNAL PRESSURE

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Let a closed circular cylindrical shell, hinged at its ends, be subjected to a uniformly distributed external pressure. We shall assume that, during deformation of the shell, the end sections remain circular and, at the same time, that the points of these sections may undergo certain radial displacements. Wishing to investigate the stability of the shell "in the large," we shall use the Ritz method and approximate the deflection function w by the expression

$$w = f(\sin \alpha x \sin \beta y + \psi \sin^2 \alpha x + \varphi); \tag{1}$$

here f is the deflection amplitude; $\alpha = \pi/L$; $\beta = n/R$; L is the length of the shell; R is the radius; n is the number of waves along the circumference; the coordinate x is measured along a generator, y along the arc; positive deflections w are directed inward into the shell. Dependences close to (1) were adopted in works ([1–4]); the first term in parentheses corresponds to the solution of the linear problem of stability "in the small," the second reflects the predominant bulging of the shell toward the center of curvature observed in experiments, and the third corresponds to the radial displacement of the points of the end sections.

Suppose that the form of the middle surface of the shell before the application of the load differs somewhat from circular, and that the law of distribution of the total deflections w_p, consisting of the initial w_n and additional w, coincides with (1):

$$w_\text{p} = w_\text{n} + w = (f_\text{n} + f)(\sin \alpha x \sin \beta y + \psi \sin^2 \alpha x + \varphi). \tag{2}$$

Under this assumption the influence of the initial imperfections is revealed especially clearly ([1]); however, only the amplitude of the initial deflection f_n may be considered prescribed. It must be taken into account that real shells, in the manufacturing process, often experience the action of such loads as, for example, impacts in the radial direction, which may lead to deformations of type (2).

The differential compatibility equation for deformations of flexible shells of medium length, taking into account initial deviations, has the form ([4])

$$\frac{1}{E}\nabla^2\nabla^2\Phi = (w_{\mathrm{p},xy})^2 - w_{\mathrm{p},xx}w_{\mathrm{p},yy} - (w_{\mathrm{n},xy})^2 + w_{\mathrm{n},xx}w_{\mathrm{n},yy} - \frac{1}{R}w_{,xx}, \tag{3}$$

where Φ is the stress function in the middle surface; indices after a comma denote differentiation with respect to x or y; ∇^2 is the Laplace operator.

Substitute (1) and (2) into the right-hand side of equation (3) and find its particular solution Φ_1. We shall write the final expression for Φ in the form $\Phi = \Phi_1 - ax^2/2$, where a is a parameter associated with the mean value of the hoop stress. The deformation along the arc ε_y is equal to

$$\varepsilon_y = v_{,y} + \frac{1}{2}(w_{,y})^2 + w_{\mathrm{n},y}w_{,y} - \frac{w}{R}. \tag{4}$$

On the other hand, within the elastic range one has

$$\varepsilon_y = \frac{1}{E}\left(\Phi_{,yy} - \mu\Phi_{,xx}\right). \tag{5}$$

Comparing (4) and (5) and taking into account the condition of closure of the shell for the circumferential displacement v,

$$\oint \frac{\partial v}{\partial y}\,dy = 0, \tag{6}$$

we find

$$\frac{a}{E} = \frac{f}{R}\left(\frac{\psi}{2} + \varphi\right) - \frac{1}{8}\left(f^2 + 2ff_{\mathrm{n}}\right)\beta^2. \tag{7}$$

Let us compute the total energy of the system

$$\mathcal{E} = U_c + U - W.$$

This includes:

the potential energy of deformation of the middle surface

$$U_c = \frac{h}{2E}\iint_F \left[(\Phi_{,xx} + \Phi_{,yy})^2 - 2(1+\mu)\left(\Phi_{,xx}\Phi_{,yy} - \Phi_{,xy}^2\right)\right] dx\,dy; \tag{8}$$

the potential energy of bending

$$U = \frac{D}{2} \iint_F \left[(w_{,xx} + w_{,yy})^2 - 2(1-\mu)\left(w_{,xx} w_{,yy} - w_{,xy}^2\right) \right] dx\, dy; \tag{9}$$

the work of the external load of intensity q

$$W = q \iint_F w\, dx\, dy; \tag{10}$$

h is the shell thickness, $F = 2\pi RL$.

Let us write the Ritz-method equations

$$\frac{\partial \mathcal{E}}{\partial \varphi} = 0, \qquad \frac{\partial \mathcal{E}}{\partial f} = 0, \qquad \frac{\partial \mathcal{E}}{\partial \psi} = 0. \tag{11}$$

The first of these, after comparison with (7), leads to the relation $a = qR/h$. The system of the two other nonlinear algebraic equations relates the deflection parameters f and ψ to the load intensity q.

Figure 1 presents the results of calculations for the case $R/h = 112.5,\ L/R = 2.2$. Along the abscissa is plotted the dimensionless deflection $\zeta = f/h$, and along the ordinate the dimensionless load $\hat{q} = qR^2/Eh^2$; $\mu = 0.3$ was adopted. The initial imperfections are characterized by the parameter $\zeta_\mathrm{n} = f_\mathrm{n}/h$. Each solid line corresponds to a definite value of ζ_n and represents the envelope of a series of curves constructed for different values of the parameter depending on the number of waves n; similar curves corresponding to a shell of circular form ($\zeta_\mathrm{n} = 0$) are shown by dashed lines in the figure. The circle on the ordinate axis marks the upper critical stress, which can also be found by means of the linear theory(5).

The deformation of a shell having one or another initial deflection must be accompanied by a snap-through if the given line in Fig. 1 has a descending segment; the latter corresponds to unstable equilibrium states. Comparison with the curve $\zeta_\mathrm{n} = 0$ shows that, in the presence of initial imperfections, the upper critical load (the highest point of the loop) decreases; this circumstance also affects the behavior of the shell in tests. However, the lowering of the upper limit is here not as sharp as under axial compression of a closed shell or of a cylindrical panel(4). Thus, for example, for $\zeta_\mathrm{n} = 0.1$ the reduction amounts, in the case of external pressure, to 12%, whereas in the case of compression along the generator it is more than 40%. The lower limit $\hat{q}$, however, remains almost constant.

If the sagitta of the initial deflection exceeds the thickness of the shell, then, judging from Fig. 1, the load changes monotonically. Let us note that, at a certain stage of loading, the "characteristic" of the shell deformation $\hat{q}(\xi)$

Fig. 1

Fig. 1

Figure 1: Fig. 1

turns out to be the steeper, the greater the value of ζ_n; the increase in stiffness is explained by the fact that, for considerable initial deflections, the shell becomes, as it were, corrugated.

These conclusions are confirmed by a series of experiments carried out by V. E. Mineev and the author on duralumin shells. Specimens that had received preliminary dents of various depths were subjected to all-round compression; the experimentally determined curves $\hat{q}(\xi)$ were located approximately as in Fig. 1.

The solution of the problem given above can be refined by increasing the number of varied parameters in an expression of type (1).

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Note: Figure translations are in progress. See original paper for figures.