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Abstract

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ON COMPRESSIONS AND DILATIONS OF ICE IN THE ARCTIC BASIN

(Presented by Academician V. V. Shuleikin, 30 IV 1957)

§ 1. Compressions and dilations under steady ice drift in the Central Arctic Basin

Previously, in determining the averaged steady drift of ice, we took into account in the equations of motion of the ice the forces due to wind, friction of the ice against the water, and the rotation of the Earth about its axis ([1]). More exact equations of motion of the ice

$$T_x + R_x + \Omega\rho'' hV = -g\rho'' h\frac{\partial\xi}{\partial x}; \qquad T_y + R_y - \Omega\rho'' hU = -g\rho'' h\frac{\partial\xi}{\partial y} \quad (1)$$

lead, in place of expressions (26) of work ([1]), to the formulas

$$U = -K\frac{\partial p}{\partial y} + K'\frac{\partial p}{\partial x} - K_g\frac{\partial(p-\Phi)}{\partial y}; \qquad V = K\frac{\partial p}{\partial x} + K'\frac{\partial p}{\partial y} + K_g\frac{\partial(p-\Phi)}{\partial x}, \quad (2)$$

in which U, V are the components of the ice-drift velocity, p is atmospheric pressure, K is the coefficient of isobaric drift, K' is the coefficient of deflected purely wind-driven drift, $K_g = (K)_{m=0}$ is the coefficient of gradient drift, and Φ is a function satisfying the Laplace equation $\Delta\Phi = 0$ with boundary condition $(\Phi)_L = (p)_L - f(L)/\rho H K_g$.

Let us represent the velocity of the total ice drift $\mathbf{V}$ as the sum of the velocities of purely wind-driven drift $\mathbf{V}_d$ and gradient drift $\mathbf{V}_g$. According to (2) we have

$$U_d = -K\frac{\partial p}{\partial y} + K'\frac{\partial p}{\partial x}; \qquad V_d = K\frac{\partial p}{\partial x} + K'\frac{\partial p}{\partial y}; \quad (3)$$

$$U_g = -K_g\frac{\partial(p-\Phi)}{\partial y}; \qquad V_g = K\frac{\partial(p-\Phi)}{\partial x}. \quad (4)$$

Formulas (2)–(4) make it possible to compute the divergence of the drift velocity

$$\operatorname{div}\mathbf{V} = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}, \tag{5}$$

which is a characteristic of compressions and dilations of the ice*. Substituting (2) into (5) and taking account of (3) and (4), we obtain

$$\operatorname{div}\mathbf{V} = \operatorname{div}\mathbf{V}_d = K'\Delta p; \tag{6}$$

$$\operatorname{div}\overline{\mathbf{V}}_g = 0. \tag{7}$$

* By compressions and dilations are meant not only local compressions and dilations, but also changes in the compactness of the ice that may occur over large areas.

From formulas (6) and (7) it is evident that the divergence of the velocity of the general drift of the ice is proportional to the Laplace operator of the atmospheric pressure, increases with an increase and decreases with a decrease in ice thickness. The gradient drift of ice takes place without any change in compactness, in contrast to a purely wind-driven drift, in which compressions and rarefactions arise. Thus, N. N. Zubov' s assertion ([2]) that compactness remains unchanged under a steady purely wind-driven ice drift is incorrect. The change in compactness in this case, as is clear from formulas (6) and (3), occurs at the expense of the deviation of the purely wind-driven ice drift from the isobaric drift*. This deviation is absent only in the limiting case when the ice thickness h decreases without bound (for $h = 0$ we have $K' = 0$ and, consequently, $\operatorname{div}\mathbf{V} = \operatorname{div}\mathbf{V}_d = 0$).

From formulas (6), (7) it follows that, in order to determine the zones of compression and rarefaction of the ice, it is sufficient to compute the Laplace operator of the atmospheric pressure. Regions in which $\Delta p < 0$ are zones of ice compression; where $\Delta p > 0$, zones of rarefaction are located; and, finally, in regions where $\Delta p = 0$, no change in the compactness of the ice occurs.

§ 2. Compressions and rarefactions under unsteady ice drift

Since the problem of determining the unsteady drift of compact ice has not been solved, at present there is no possibility of directly calculating the divergence of the velocity of unsteady drift, which characterizes compressions and rarefactions of the ice. However, some idea of the compressions and rarefactions of ice under unsteady purely wind-driven drift can be obtained by analyzing the field of tangential wind stress, whose components are determined by the formulas

$$T_x = -\sqrt{\frac{A'}{2\Omega}\left(\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\right)}; \qquad T_y = \sqrt{\frac{A'}{2\Omega}\left(\frac{\partial p}{\partial x} - \frac{\partial p}{\partial y}\right)}, \tag{8}$$

in which A' is the coefficient of vertical exchange in the atmosphere and Ω is the Coriolis parameter ([1]).

This analysis is based on the following two considerations:

1. A purely wind-driven drift of ice changes rapidly when the wind changes (several hours of wind from a new direction are sufficient for the drift velocity to change completely both in magnitude and in direction ([3])).
2. During the development of a purely wind-driven drift, ice floes tend to move along the forces acting on them, i.e. in the direction of the wind, just as during the development of a purely drift current at the sea surface free of ice ([4]).

From these considerations it follows that, under rapidly changing wind conditions, one may expect compression of the ice where there is convergence of the forces acting on the ice from the atmosphere ($\operatorname{div}\mathbf{T} < 0$), and rarefaction where there is divergence of these forces ($\operatorname{div}\mathbf{T} > 0$).

Expressing the divergence of the tangential wind stress through atmospheric pressure with the aid of formulas (8), in which the exchange coefficient and the Coriolis parameter are assumed constant:

$$\operatorname{div}\mathbf{T} = -\sqrt{\frac{A'}{2\Omega}}\,\Delta p, \tag{9}$$

we obtain that in regions where $\Delta p > 0$ there is a tendency toward compression, while in regions where $\Delta p < 0$ there is a tendency toward rarefaction of the ice. This conclusion is opposite to the result obtained in § 1 for a steady purely wind-driven drift of ice.

* A purely wind-driven drift of ice deviates from the isobaric drift to the right by an angle whose tangent, as follows from (3), is equal to K'/K. For $K' = 0.2K$ this angle is $\sim 11°$.

The indicated contradiction in the results obtained is easy to explain if one takes into account the simple relation between $\operatorname{div}\mathbf{T}$ and the vertical components of the curls of the tangential wind-stress and geostrophic-wind velocity vectors $\mathbf{W}$*

$$\operatorname{div}\mathbf{T} = -\operatorname{rot}_z\mathbf{T} = \rho'\sqrt{\frac{\Omega A'}{2}}\,\operatorname{rot}_z\mathbf{W}. \tag{10}$$

Thus, for example, in a region where the atmospheric circulation has a cyclonic character ($\operatorname{rot}\mathbf{W} = \Delta p/\Omega\rho' > 0$), in the stationary case we have a rarefaction of the ice owing to the deviation of the purely wind-driven ice drift to the right of the isobars, due to the finite thickness of the ice; and in the nonstationary case, a compression of the ice owing to the convergence of the forces acting on the ice.

Let us suppose that at the initial moment a cyclone arose over motionless, compact ice, and that it subsequently does not change with time. At first there will be compression of the ice, which in its drift deviates to the left of the isobars. Then, as the current beneath the ice develops, the ice drift will become isobaric, after which the ice will begin to diverge, deviating to the right of the isobars.

The example considered is highly idealized, since under the real conditions of the Arctic Basin the wind is changing all the time. The question arises as to in what cases one may judge compressions and rarefactions of the ice on the basis of a scheme constructed for unsteady drift, and when one may determine zones of compression and rarefaction by using the theory of established ice drift. Apparently, under rapid changes in wind conditions the first scheme will predominate, while under a slowly varying wind field—the second. A definitive answer to the question of choosing the first or the second scheme in solving concrete practical problems will be possible only after a careful analysis of observational materials both on local compressions and rarefactions of the ice and on changes in ice compactness occurring over large areas of the Arctic Basin.

§ 3. Index of Atmospheric Circulation

As some preliminary calculations by A. L. Sokolov have shown, the considerations presented in the preceding paragraph may be used in establishing the relation between ice conditions in the marginal seas and atmospheric circulation in the Central Arctic, and also in developing methods for short-range and long-range forecasts of local ice compressions and rarefactions. In this connection it seems expedient to introduce an index of atmospheric circulation, defining it at a point $A(x, y)$ as a quantity proportional to the divergence of the vector of the atmospheric-pressure gradient:

$$I_A = k \operatorname{div} \operatorname{grad} p = k \, \Delta p. \tag{11}$$

In addition, one may introduce an integral index of atmospheric circulation over an area σ:

$$I_\sigma = k \iint_\sigma \Delta p \, dx \, dy, \tag{12}$$

and also with respect to time from t_1 to t_2:

$$I_{A,\,t_2-t_1} = \int_{t_1}^{t_2} I_A\,dt; \tag{13}$$

$$I_{\sigma,\,t_2-t_1} = \int_{t_1}^{t_2} I_\sigma\,dt. \tag{14}$$

* The divergence and curl of the tangential wind stress are also related when the exchange coefficient in the atmosphere A' varies with height. In this case $\operatorname{div}\mathbf{T} = -c\,\operatorname{rot}_z\mathbf{T}$, where c is a certain positive constant.

If, in solving practical problems, one uses a square kilometer grid with constant spacing h, it is convenient to choose $k = \frac{1}{h^2}$, since in this case the index of atmospheric circulation at the grid point A is determined simply as the sum of the atmospheric-pressure values at the four neighboring grid points minus four times the value of the atmospheric pressure at point A.

In establishing empirical relationships between ice concentration, which characterizes the areal distribution of ice, and atmospheric circulation, pressure gradients ([5], [6]), which are not related to area, are usually used as indicators of atmospheric circulation. The relationships obtained in this way are poorly justified physically. The index we propose, however, is not only related to area but also has a direct physical meaning, characterizing the nonuniformity of the forces acting on the ice from the atmosphere. Therefore, when it is used, apparently more reliable relationships can be obtained. Let us note that, in addition to the index proposed here, it is also possible to use the atmospheric-circulation index of N. A. Belinskii ([7]).

The considerations set forth in § 3 arose during a discussion of questions of the methodology of ice forecasting with A. L. Sokolov, to whom the author expresses his gratitude.

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Note: Figure translations are in progress. See original paper for figures.