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HYDROMECHANICS

G. G. CHERNYI

1957

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Abstract

Full Text

HYDROMECHANICS

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FLOW PAST A THIN BLUNTED CONE AT HIGH SUPERSONIC VELOCITY

(Presented by Academician L. I. Sedov on 7 VI 1957)

In work ([1]) we formulated the general problem of the flow past thin bodies of revolution or profiles, blunted at the front, by a gas stream at high supersonic velocity, and obtained an approximate solution for the case of flow past a blunted wedge. Below, in the same approximate formulation, a solution is given for the problem of flow past a blunted cone, and the solution found is compared with experimental data.

Let us consider the symmetric flow of a gas stream with velocity V past a thin blunted cone with semi-angle α. In accordance with the basic hypothesis of work ([1]), we replace the action of the blunted part of the cone on the gas by the action of a concentrated force X—the drag of the bluntness—applied to the gas from the side of the bluntness, and we shall neglect the size of the latter. Using the law of plane sections ([2]), we reduce the problem of flow at high supersonic velocity past a blunted cone to the following equivalent problem of one-dimensional unsteady motion of a gas with cylindrical waves (the coordinate x along the flow must in this case be replaced by Vt, where t is time). At the initial instant, on a straight line located in a stationary gas, energy E is released (per unit length of the straight line; here $E = X$), and the gas begins to expand as a cylindrical piston whose radius grows proportionally to time with velocity $U = V \tan\alpha$; in addition, in the general case the gas particles acquire an initial impulse I (per unit layer of unit width and angle 2π) in the radial direction. For $E \neq 0$ and $I \neq 0$, and also when the initial pressure is taken into account, the resulting motion is not self-similar, and an exact solution of the formulated problem can be obtained only by complicated numerical methods (see, for example, ([3])). In an approximate solution of the problem we shall assume that the entire mass of gas in the disturbed region is concentrated on the arising shock wave and moves together with it. Then, denoting by R the radius of the cylindrical shock wave, and by ρ_0 and p_0 the initial density and pressure of the gas, we write the equations of conservation of momentum and energy for the gas in the disturbed region in the form

Fig. 1. a—experiment, $K = 1.2$; b—theory, $K = \infty$

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Figure 1: a –experiment, $K = 1.2$; –theory, $K = \infty$

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Fig. 2

Figure 2: Fig. 2

$$\pi\rho_0 R^2 \dot{R}^2 = I + 2\pi \int_0^t (p - p_0) R \, dt,$$

$$\pi\rho_0 R^2 \frac{\dot{R}^2}{2} + \pi(R^2 - U^2 t^2) \frac{p}{\gamma - 1} = E + \pi R^2 \frac{p_0}{\gamma - 1} + 2\pi U^2 \int_0^t pt \, dt. \quad (1)$$

Here p is the gas pressure in the disturbed region; γ is the ratio of specific heats; the dot over R denotes differentiation with respect to time. These two equations make it possible, for given $\rho_0, p_0, \gamma, E, I, U$, to determine the time dependence of the quantities p and R and, consequently, returning to the problem

Fig. 2. *a* –experiment, $K = 1.2$; *b* –theory, $K = \infty$

of the flow past a blunted cone, make it possible to determine the shape of the shock wave that arises and the pressure distribution over the cone.

Leaving aside the case $U = 0$, which was considered by the author earlier in the formulation presented here ([4]), we introduce, for measuring length, the scale

$$L = (E/\pi\rho_0 U^2)^{1/2},$$

and for measuring time, the scale L/U, and denote

$$p - p_0 = \rho_0 U^2 p.$$

Then equations (1) take the form (the bar over p is omitted):

$$R^2 \ddot{R} = \frac{IU}{E} + 2 \int_0^t pR \, dt,$$

$$\frac{R^2 \dot{R}^2}{2} + \frac{R^2 - t^2}{\gamma - 1} \left(p + \frac{1}{\gamma K^2} \right) = 1 + \frac{R^2}{\gamma - 1} \frac{1}{\gamma K^2} + 2 \int_0^t \left(p + \frac{1}{\gamma K^2} \right) t \, dt. \quad (2)$$

Here

$$K^2 = \frac{U^2}{\gamma p_0 / \rho_0}.$$

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Fig. 3

Figure 3: Fig. 3

For small values of t, the initial energy of the gas in the disturbed region and the work performed by the piston are small in comparison with the energy released in the explosion, and the solution of system (2) passes over into the approximate solution of the problem of a strong explosion with cylindrical waves ([5])

$$R = \left[\frac{8(\gamma-1)}{\gamma}\right]^{1/4} t^{1/2}, \qquad p = \left(\frac{\gamma-1}{8\gamma}\right)^{1/2} t^{-1}. \tag{3}$$

In this case the first of equations (2) is satisfied for small t only if in it $IU/E = 0$. Since for a thin cone the quantity IU/E is proportional to $\operatorname{tg}\alpha$ and, consequently, is small, the solution which, for small t,

the asymptotic form (3), can be used to estimate the influence of blunting of the cone tip on its flow in a stream with high supersonic velocity. For large values of t the function R must tend to a linear function, and the functions $\dot{R}$ and p to constants corresponding to the flow past a pointed cone.

The solution of system (2) having the asymptotic form (3) can be obtained by numerical integration. Calculations carried out for the case $K = \infty$ (i.e., neglecting the initial pressure) revealed the following interesting features of the behavior of the solution*. The pressure coefficient on the cone, equal to infinity at the front point, rapidly decreases when moving along the generator of the cone, taking on some segment values substantially smaller than the values on a pointed cone with the same angle of aperture (see Fig. 1). Correspondingly, the angle β between the shock wave and the direction of the flow also has a minimum (Fig. 2). In Figs. 1 and 2, experimental data are also given, obtained [(6)] for the flow past a cone with a semi-angle of aperture 10° and with different blunting at $M = 6.85$. The qualitative agreement of the results of theory ($K = \infty$) and experiment ($K = 1.2$) is unquestionable.

Fig. 3

Since the pressure over a considerable part of the surface of the blunt cone is lower than on the surface of a sharp cone, the total drag of the blunt cone may turn out to be lower than the drag of the sharp cone. In Fig. 3 are shown the values of the drag coefficient of the cone $C_x^{(k)}$, calculated by the formula ($K = \infty$):

$$C_x^{(k)} = \frac{C_x}{\operatorname{tg}^2\alpha}\frac{d^2}{l^2}\left[1 + 2\int_0^{\sqrt{\frac{2}{C_x}\operatorname{tg}^2\alpha\,\frac{l}{d}}} pt\,dt\right].$$

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Here d and l are, respectively, the diameter of the blunting and the length of the cone;

$$C_x = \frac{2X}{\rho_0 V^2 \pi d^2/4}$$

is the drag coefficient of the blunting. The drag of the blunt cone has a minimum at

$$\frac{l}{d} \simeq \frac{0.88}{\operatorname{tg}^2 \alpha} \sqrt{\frac{C_x}{2}},$$

and the relative decrease of the drag coefficient in comparison with the sharp cone reaches 10%. Also plotted in Fig. 3 (dashed line) are the values of $C_x^{(k)}$ obtained by simply adding the drag of the sharp cone and the drag of the blunting.

Received
4 VI 1956

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* The calculations were performed by P. Krasnoshchekov, to whom the author expresses sincere gratitude.

Note: Figure translations are in progress. See original paper for figures.

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