Academician N. N. BOGOLYUBOV and D. V. SHIRKOV

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Abstract

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Academician N. N. BOGOLYUBOV and D. V. SHIRKOV

DISPERSION RELATIONS FOR COMPTON SCATTERING ON NUCLEONS

In view of the importance of the question of experimental verification of the basic propositions of quantum field theory, much attention is now being paid to dispersion relations, chiefly to dispersion relations for meson-nucleon scattering. In our opinion, dispersion relations for the Compton effect on nucleons, which from the theoretical point of view have a number of advantages, are also of definite interest*.

Below, in analyzing the amplitude of Compton scattering f, we shall restrict ourselves to considering its principal term in e, proportional to e^2, and therefore in the expressions for the corresponding variational derivatives we shall set $e = 0$, thereby taking into account only the strong interactions.

Dispersion relations for the scattering of photons on nucleons can be obtained by the same method that was proposed by one of the authors ([2]) for obtaining dispersion relations for the scattering of π-mesons on nucleons.

We write the amplitude of Compton scattering in the form

$$f^c_{A\Omega}(\mathbf{k}, \mathbf{k}') = \sum_{n,n'} e^n_\nu(\mathbf{k})\, e^{n'}_{\nu'}(\mathbf{k}')\, T^c_{\alpha\omega}\left(\frac{k+k'}{2}\right). \tag{1}$$

Here $\mathbf{k}$ is the momentum of the initial photon; $A = (\mathbf{p}, s, \nu)$, where $\mathbf{p}$ is the initial momentum of the nucleon; s are the discrete initial indices of the nucleon; ν is the polarization index of the initial photon. The final state is described by primed quantities. In this notation $\Omega = (\mathbf{p}', s', \nu')$, $\alpha = (\mathbf{p}, s, n)$, and $\omega = (\mathbf{p}', s', n')$.

The function T^c is the momentum representation of the "causal" matrix element

$$T^c_{\alpha\omega}(k) = \int e^{ikx} F^c_{\alpha\omega}(x)\, dx; \tag{2}$$

$$F^c_{\alpha\omega}(x-y) = \frac{2\pi^2}{i}\, e^{\frac{i(p-p')(x+y)}{2}} \left\langle \mathbf{p}'\, s' \left| \frac{\delta^2 S}{\delta A_{n'}(x)\, \delta A_n(y)}\, S^\dagger \right| \mathbf{p}\, s \right\rangle. \tag{3}$$

Introducing further the "retarded" and "advanced" matrix elements

$$F^{\rm ret}_{\alpha\omega}(x-y) = \frac{2\pi^2}{i}\, e^{\frac{i(p-p')(x+y)}{2}} \left\langle \mathbf{p}'\, s' \left| \frac{\delta}{\delta A_{n'}(x)} \left(\frac{\delta S}{\delta A_n(y)}\, S^\dagger \right) \right| \mathbf{p}\, s \right\rangle; \qquad (4)$$

$$F^{\rm adv}_{\alpha\omega}(x-y) = P_{nn'}\, F^{\rm ret}_{\omega\alpha}(y-x) \qquad (5)$$

* A number of very interesting considerations and results concerning dispersion relations for forward Compton scattering are contained in work ([1]).

and, carrying out the usual reasoning in the coordinate system $\mathbf{p}+\mathbf{p}'=0$, where $k^0={k'}^0=E$, $\mathbf{k}=\lambda\vec{\eta}-\mathbf{p}$, $\mathbf{k}'=\lambda\vec{\eta}+\mathbf{p}$, $\vec{\eta}^{\,2}=1$, $\lambda=\sqrt{E^2-\mathbf{p}^2}$, we obtain the possibility of determining, for the fictitious case $\lambda^2=E^2-\mathbf{p}^2-\tau$, $\tau<-\mathbf{p}^2$, the functions $S_\pm\tilde{f}$:

$$S_\pm \tilde{f}(E;\mathbf{k},\mathbf{k}') = \frac{\tilde{f}_{A\Omega}(E,\lambda\vec{\eta}) \pm \tilde{f}_{A\Omega}(E,-\lambda\vec{\eta})}{\frac{1\mp 1}{2}\, 2\lambda^2}; \qquad (6)$$

$$\tilde{f}(E,\lambda\vec{\eta}) = \begin{cases} f^{\rm ret}_{A\Omega}(E,\lambda\vec{\eta}), & \operatorname{Im} E>0; \\ f^{\rm adv}_{A\Omega}(E,\lambda\vec{\eta}), & \operatorname{Im} E<0; \end{cases} \qquad (7)$$

$$\lim_{\varepsilon\to+0} \tilde{f}(E,\lambda\vec{\eta}) = f^{\rm ret}(\operatorname{Re} E,\lambda\vec{\eta}) \quad \text{for } E=\operatorname{Re} E+i\varepsilon, \quad \operatorname{Re} E>|\mathbf{p}|, \qquad (8)$$

analytic in the whole plane of the complex variable E, except for cut lines and poles located on the real axis. These cut lines and poles are determined by expanding in a complete system of functions of the Hamiltonian of the meson and nucleon fields. In this way we obtain, for real E:

$$Sf(E) = Sf^{\rm ret}(E) - Sf^{\rm adv}(E) = Sf^{(+)}(E) - Sf^{(-)}(E) =$$

$$= \frac{(2\pi)^6}{2i} \sum_N \delta\left(\sqrt{\mathbf{q}^2+M_n^2} + q^0 - \frac{p^0+p'^0}{2} \right) S \langle \mathbf{p}'s' | J_n(0) | -\mathbf{q}N \rangle \times$$

$$\times \langle -\mathbf{q}N | J_{n'}(0) | \mathbf{p}s \rangle - \frac{(2\pi)^6}{2i} S \sum_N \delta\left(\sqrt{\mathbf{q}^2+M_n^2} - q^0 - \frac{p^0+p'^0}{2} \right) \langle \mathbf{p}'s' | J_{n'}(0) | \mathbf{q}N \rangle \langle \mathbf{q}N | J_n(0) | \mathbf{p}s \rangle. \qquad (9)$$

Here $q=\{E,\lambda\vec{\eta}\}$, $J_n(x)=i\dfrac{\delta S}{\delta A_n(x)}S^+$, and the summation over N extends over all discrete and continuous parameters of the complete system of functions $|N\rangle$. Assuming that the system nucleon + photon has no bound states with mass $M_n<M+\mu$, except for the one-nucleon state $M_n=M$, we find that the

Fig. 1

Figure 1: Fig. 1

functions $S\tilde{f}$ have, on the real axis, two poles at $E = \pm E_p$ and two cut lines $(-\infty, -E_1)$, (E_1, ∞). The expressions for E_p and E_1 (for $\tau = 0$) have the form

Fig. 1

$$E_p = \frac{\mathbf{p}^2}{\sqrt{M^2 + \mathbf{p}^2}}; \qquad E_1 = \frac{2M\mu + \mu^2 - 2\mathbf{p}^2}{2\sqrt{M^2 + \mathbf{p}^2}}. \tag{10}$$

At the same time $E_1 > E_p$ in the momentum region where

$$\mathbf{p}^2 < \frac{M\mu}{2} + \frac{\mu^2}{4}. \tag{11}$$

Assuming further that the Compton-scattering amplitude has, at infinity, a pole of no higher than first order,* we may use the Cauchy integral formula for the functions $S\tilde{f}(E)/(E - E_0)^2$, where E_0 is a real parameter such that $|E_0| < E_1$, $|E_0| \neq E_p$. The integration contour has the form shown in Fig. 1. Letting the ra–

* It presents no difficulty to consider other possibilities as well.

radius of the large semicircles to infinity and that of the small ones to zero, we obtain the equations

$$S\tilde{f}(E) = \frac{(E - E_0)^2}{2\pi i} \int_{|E'|>E_1} \frac{Sf(E')\,dE'}{(E' - E_0)^2 (E' - E)} + \left(\frac{E_0 - E}{E_0 - E_p}\right)^2 \frac{Sf_{\nu\nu'}(-\lambda\vec{\eta})}{E - E_p} - \left(\frac{E_0 - E}{E_0 + E_p}\right)^2 \frac{Sf_{\nu'\nu}(\lambda\vec{\eta})}{E + E_p} + c_0 + c_1 \tag{12}$$

Here $f_{\nu\nu'}$ and $f_{\nu'\nu}$ are the "intensities" of the one-nucleon terms from (9), which, after carrying out the analytic continuation to $\tau = 0$, take the form (cf. the unobservable term in the dispersion relations for photoproduction (3)):

$$f_{\nu\nu'}(\lambda\vec{\eta}) = \frac{e^2}{4\pi} \frac{1}{\sqrt{M^2 + \mathbf{p}^2}} \bar{v}^+_{s'}(\mathbf{p}') \left\{ \hat{e}_\nu(\mathbf{k}')\,\tau_P + \frac{a + b\tau_3}{2} \frac{\hat{e}_\nu(\hat{p}' - \hat{p}'') - (\hat{p}' - \hat{p}'')\hat{e}_{\nu'}}{2} \right\} \times$$

$$\times (\hat{p}'' + M) \left\{ \hat{e}_\nu(\mathbf{k})\,\tau_P + \frac{a + b\tau_3}{2} \frac{\hat{e}_\nu(\hat{p}'' - \hat{p}) - (\hat{p}'' - \hat{p})\hat{e}_\nu}{2} \right\} v^-_s(\mathbf{p}). \tag{13}$$

In (13), p'' is the 4-momentum of the intermediate nucleon,

$$\mathbf{p}'' = \lambda_p \vec{\eta}, \qquad p_0'' = \sqrt{\lambda_p^2 + M^2} = \frac{M^2}{\sqrt{M^2 + \mathbf{p}^2}};$$

$$\frac{e^2}{4\pi} = \frac{1}{137}, \qquad a = \frac{\mu_P - \mu_N}{2M}, \qquad b = \frac{\mu_P - \mu_N}{2M}, \qquad \mu_P = 1.7896, \qquad \mu_N = -1.9103.$$

In taking in (12) the limiting transition to real E, we shall use the fact that

$$^+f^{\text{ret}}_{\Lambda\Omega}(\mathbf{k}, \mathbf{k}'; E) \equiv {}^*f^{\text{ret}}_{\Omega\Lambda}(\mathbf{k}', \mathbf{k}; E) = f^{\text{adv}}_{\Lambda\Omega}(\mathbf{k}, \mathbf{k}'; E). \tag{14}$$

This makes it possible to decompose the observable amplitude $f^c(E)$, for $\operatorname{Im} E = 0$, $|\operatorname{Re} E| > |\mathbf{p}|$, into its Hermitian and anti-Hermitian parts:

$$Sf^c(E) = Sf^{\text{ret}}(E) = D_{\Lambda\Omega}(E) + iA_{\Lambda\Omega}(E), \tag{15}$$

where

$$D = \frac{Sf^{\text{ret}} + Sf^{\text{adv}}}{2} = S\tilde{f} - \frac{1}{2}Sf,$$

$$iA = \frac{Sf^{\text{ret}} - Sf^{\text{adv}}}{2} = \frac{Sf}{2}.$$

To exclude the unobservable region of negative energies, it is necessary to take into account the symmetry properties of the functions D and A with respect to the variable E. Using the symmetrization and antisymmetrization operations $(1 + p_{\nu\nu'})$ over the spin indices ν and ν', after simple transformations we obtain the following dispersion relations:

$$D_{\text{even}}(E) - D_{\text{even}}(E_0) = \frac{2(E^2 - E_0^2)}{\pi} P \int_{E_1}^{\infty} \frac{E' A_{\text{odd}}(E')\, dE'}{({E'}^2 - E_0^2)({E'}^2 - E^2)} +$$

$$+ \frac{E^2 - E_0^2}{E_p^2 - E_0^2} \frac{E_p f_{\text{odd}}}{E_p^2 - E^2}; \tag{16}$$

$$D_{\text{odd}}(E) - \frac{E}{E_0} D_{\text{odd}}(E_0) = \frac{2E(E^2 - E_0^2)}{\pi} P \int_{E_1}^{\infty} \frac{A_{\text{even}}(E')\, dE'}{({E'}^2 - E_0^2)({E'}^2 - E^2)} +$$

$$+ \frac{E^2 - E_0^2}{E_p^2 - E_0^2} \frac{E f_{\text{even}}}{E_p^2 - E^2}. \tag{17}$$

Here

$$D_{\text{even/odd}}(E) = \{(1 \pm p_{\nu\nu'})S_+ D_{A\Omega}(E); \quad (1 \mp p_{\nu\nu'})S_- D_{A\Omega}(E)\}; \tag{18}$$

$$A_{\text{odd/even}}(E) = \{(1 \pm p_{\nu\nu'})S_+ A_{A\Omega}(E); \quad (1 \mp p_{\nu\nu'})S_- A_{A\Omega}(E)\}; \tag{19}$$

$$f_{\text{odd/even}} = \{(1 \pm p_{\nu\nu'})S_+ f_{\nu'\nu}(\lambda, \vec{p}\,\eta); \quad (1 \mp p_{\nu\nu'})S_- f_{\nu'\nu}(\lambda, \vec{p}\,\eta)\}. \tag{20}$$

An important property of the dispersion relations obtained, (16) and (17), is the absence in them of an unobservable energy region not only for forward scattering, but also for a finite interval of recoil momenta

$$|\mathbf{p}| < \mu\,\frac{M + \mu/2}{M + \mu}. \tag{21}$$

It is also interesting to note that in the energy interval

$$E_1 < |E'| < E_2 = \frac{2M\mu + 2\mu^2 - \mathbf{p}^2}{\sqrt{\mathbf{p}^2 + M^2}} \tag{22}$$

the integrand can be expressed in terms of the amplitudes of the photoproduction process. This is connected with the fact that in the interval (22), in the sum (9), only the term corresponding to a state containing a nucleon + meson differs from zero. A detailed consideration of this term leads us to the formula

$$Sf(E) = \frac{i}{2\rho k^0}\sum_{\rho,s''} S\int dl\,\delta\Big(\sqrt{(1-\lambda\vec{\eta})^2 + M^2} + \sqrt{l^2+\mu^2} - E - \sqrt{\mathbf{p}^2+M^2}\Big)\times$$

$$\times\bar{\varphi}_{\Omega'\Sigma}\left(\frac{\lambda\vec{\eta} + 1 - \mathbf{p}'}{2}\right)\varphi_{\Sigma A}\left(\frac{\lambda\vec{\eta} + 1 - \mathbf{p}}{2}\right) -$$

$$-\frac{i}{2\pi k^0}\sum_{\rho,s''} S\int dl\,\delta\Big(\sqrt{(\lambda\vec{\eta}+1)^2 + M^2} + \sqrt{l^2+\mu^2} +$$

$$+ E - \sqrt{\mathbf{p}^2+M^2}\Big)\,\bar{\varphi}_{\Omega\Sigma}\left(\frac{1 \mp \lambda\vec{\eta} - \mathbf{p}'}{2}\right)\varphi_{\Sigma A'}\left(\frac{1 - \lambda\vec{\eta} - \mathbf{p}}{2}\right). \tag{23}$$

Here $A = (\mathbf{p}, s, \nu)$; $A' = (\mathbf{p}, s, \nu')$; $\Omega = (\mathbf{p}', s', \nu)$; $\Omega' = (\mathbf{p}', s', \nu')$; $\Sigma = (\mathbf{p}'', s'', \rho)$; $\mathbf{p}'' = -\mathbf{l} \pm \lambda\vec{\eta}$; $\varphi_{\Sigma A}\left(\frac{l+\mathbf{k}}{2}\right)$ is the amplitude of the process of photoproduction of a meson $(\mathbf{l}, \rho)$ by a photon $(\mathbf{k}, \nu)$, linearly dependent on the polarization vector $\mathbf{e}(\mathbf{k})$; $\bar{\varphi}_{\Omega\Sigma}$ is the amplitude of the inverse photoproduction process.

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REFERENCES

1. M. Gell-Mann, M. Goldberger, W. Thirring, Phys. Rev., **95**, 1612 (1954); M. Gell-Mann, M. Goldberger, Proc. 1954 Glasgow Conference on Nucl. and Meson Physics, London, 1955.

2. N. N. Bogolyubov, Report at the International Conference of Theoretical Physicists in Seattle, September 1956; N. N. Bogolyubov, B. V. Medvedev, M. K. Polivanov, *Problems in the Theory of Dispersion Relations*, 1957; N. N. Bogolyubov, D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, Ch. 9, 1957.

3. E. Corinaldesi, Nuovo Cimento, **4**, 1384 (1956); A. A. Logunov, B. M. Stepanov, A. N. Tavkhelidze, DAN, **112**, 45 (1957).

Note: Figure translations are in progress. See original paper for figures.