S. V. STRAKHOV

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

ELECTRICAL ENGINEERING

S. V. STRAKHOV

ON THE INFLUENCE OF SECOND HARMONICS IN THE EXPANSIONS OF THE INDUCTANCES AND MUTUAL INDUCTANCES OF THE STATOR ON THE STRUCTURE OF THE EQUATIONS OF TRANSIENT PROCESSES OF A SYNCHRONOUS MACHINE

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As is known, the equations of transient electromechanical processes of a salient-pole synchronous machine, written in phase coordinates, are very inconvenient for carrying out investigations, since the equations relating the flux linkages of the stator and rotor to the currents, and the equation of the electromagnetic torque, contain periodic functions of the angle

$$\theta = \int_0^t \omega \, dt + \theta_0$$

between the magnetic axis of stator phase a and the longitudinal axis d of the rotor (Fig. 1).

Park ([1]) proposed making in the above equations such a change of variables as makes it possible to eliminate from them the periodic functions of the angle θ. In this case, when the rotor speed is constant, these equations are transformed into equations with constant coefficients, which greatly facilitates the investigation of transient electromagnetic processes in a synchronous machine.

Fig. 1

From the geometrical point of view, the Park transformation is usually interpreted as referring the equations of a synchronous machine to coordinate axes

Fig. 2

Figure 2: Fig. 2

rotating with the same speed as the rotor (in other words, to axes rigidly connected with the rotor).

However, as will be shown below, referring the equations to coordinate axes rigidly connected with the rotor is still insufficient for eliminating the periodic coefficients from the original equations of the machine. It is also necessary to impose an additional relation on the coefficients in the Fourier-series expansions of the inductances and mutual inductances of the stator. Only when this additional relation is satisfied can the periodic coefficients be eliminated. Therefore, the coordinate transformation, generally speaking, is not equivalent to replacing all circuits or parts of them by transformed circuits rotating together with the chosen coordinate axes ([2]).

Fig. 2

To prove our assertion, let us consider a synchronous machine connected to a network (Fig. 2), whose phase voltages u_{ca},

u_{cb}, u_{cc}; phase currents of the stator i_{ca}, i_{cb}, i_{cc}; voltages at the terminals of the field winding u_f, of the direct-axis damper winding u_g, and of the quadrature-axis damper winding u_h; the currents of these windings, respectively, i_f, i_g, and i_h. The arrangement of the windings is shown in Fig. 1. The positive directions of the currents with respect to the like-named terminals of the windings are shown in Fig. 3.

Let us write the equations of Ohm' s law and of the flux linkages of the stator phases and rotor windings:

$$[u_c] = r_c[i_c] + \frac{d[\psi_c]}{dt}; \qquad [u_p] = [R_{pp}][i_p] + \frac{d[\psi_p]}{dt}; \tag{1}$$

$$[\psi_c] = [L_{cc}][i_c] + [M_{cp}][i_p]; \qquad [\psi_p] = [M_{pc}][i_c] + [L_{pp}][i_p],$$

where

$$[u_c] = \begin{bmatrix} u_{ca} \\ u_{cb} \\ u_{cc} \end{bmatrix}; \qquad [u_p] = \begin{bmatrix} u_f \\ u_g \\ u_h \end{bmatrix};$$

$$[R_{pp}] = \begin{bmatrix} r_f & 0 & 0 \\ 0 & r_g & 0 \\ 0 & 0 & r_h \end{bmatrix}; \qquad [L_{pp}] = \begin{bmatrix} L_f & M_{fg} & 0 \\ M_{fg} & L_g & 0 \\ 0 & 0 & L_h \end{bmatrix}; \tag{2}$$

Fig. 3

$$[L_{cc}] = \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ab} & L_b & M_{bc} \\ M_{ac} & M_{bc} & L_c \end{bmatrix} = \tag{3}$$

$$= \begin{bmatrix} L_{cp} + L_m \cos 2\theta & -M_{cp} + M_0 \cos(2\theta - 120^\circ) & -M_{cp} + M_0 \cos(2\theta + 120^\circ) \\ -M_{cp} + M_0 \cos(2\theta - 120^\circ) & L_{cp} + L_m \cos(2\theta + 120^\circ) & -M_{cp} + M_0 \cos 2\theta \\ -M_{cp} + M_0 \cos(2\theta + 120^\circ) & -M_{cp} + M_0 \cos 2\theta & L_{cp} + L_m \cos(2\theta + 120^\circ) \end{bmatrix};$$

$$[M_{cp}] = [M_{pc}]^t = \begin{bmatrix} M_f \cos\theta & M_g \cos\theta & -M_h \sin\theta \\ M_f \cos(\theta - 120^\circ) & M_g \cos(\theta - 120^\circ) & -M_h \sin(\theta - 120^\circ) \\ M_f \cos(\theta + 120^\circ) & M_g \cos(\theta + 120^\circ) & -M_h \sin(\theta + 120^\circ) \end{bmatrix}. \tag{4}$$

Here L_f, L_g, L_h are the inductances of the field winding and of the direct- and quadrature-axis damper windings; M_{fg} is the mutual inductance of the field winding and the direct-axis damper winding; M_f, M_g, M_h are the maximum values of the mutual inductance of a stator phase with the field winding and with the direct- and quadrature-axis damper windings.

We introduce the stator $[A_c]$ and rotor $[A_p]$ transformation matrices:

$$[A_c] = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ -\sin\theta & -\sin(\theta - 120^\circ) & -\sin(\theta + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}; \qquad [A_p] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}. \tag{5}$$

These matrices make it possible to relate the values of the voltages, currents, and flux linkages of the stator and rotor before and after the transformation. We shall denote the matrices of voltages, currents, and flux linkages after transformation by a prime index:

$$[u'_c] = \begin{bmatrix} u_{cd} \\ u_{cq} \\ u_{c0} \end{bmatrix} = [A_c][u_c]; \qquad [u'_p] = \begin{bmatrix} u_{pd} \\ u_{pq} \\ u_{p0} \end{bmatrix} = [A_p][u_p]. \tag{6}$$

Let us now abandon the assumption that has usually been made up to now in the literature ([3–5]), namely, let us not assume that the amplitudes of the second harmonics L_m and M_0 of the Fourier expansions of the inductances L_a, L_b, L_c and mutual inductances M_{ab}, M_{ac}, M_{bc} of the stator windings are identical. On the contrary, let us assume that they are not identical, i.e.

$$L_m \neq M_0. \tag{7}$$

To transform the equations of Ohm' s law and of the stator flux linkages, we multiply on the left both parts of (1) for $[u_c]$ and $[\psi_c]$ by $[A_c]$. Taking (2) and (6) into account, we obtain

$$[u'_c] = r_c[i'_c] + \frac{d[\psi'_c]}{dt} + \begin{bmatrix} -\psi_{cq} \\ \psi_{cd} \\ 0 \end{bmatrix} \frac{d\theta}{dt}; \qquad [\psi'_c] = [L'_{cc}][i'_c] + [M'_{cp}][i'_p], \tag{8}$$

where

$$[L'_{cc}] = [A_c][L_{cc}][A_c^{-1}] =$$

$$= \begin{bmatrix} L_d + 0.5(L_m - M_0) & 0 & (L_m - M_0)\cos 3\theta \\ 0 & L_q - 0.5(L_m - M_0) & -(L_m - M_0)\sin 3\theta \\ 0.5(L_m - M_0)\cos 3\theta & -0.5(L_m - M_0)\sin 3\theta & L_0 \end{bmatrix}. \tag{9}$$

$[A_c^{-1}]$ and $[A_p^{-1}]$ are the inverse matrices of the stator and rotor transformations; $L_0 = L_{cp} - 2M_{cp}$ is the stator zero-sequence inductance; $L_d = L_{cp} + M_{cp} + \frac{3}{2}M_0$, $L_q = L_{cp} + M_{cp} - \frac{3}{2}M_0$ are the synchronous stator inductances along the direct and quadrature axes. Writing (8) in expanded form, we obtain the equalities given by A. G. Iosifyan ([6]).

To transform the equations of Ohm' s law and of the rotor flux linkages, we multiply on the left both parts of (1) for $[u_p]$ and $[\psi_p]$ by $[A_p]$. Taking (6) into account, we obtain

$$[u'_p] = [R'_{pp}][i'_p] + \frac{d[\psi'_p]}{dt}; \qquad [\psi'_p] = [M'_{pc}][i'_c] + [L'_{pp}][i'_p], \tag{10}$$

where $[M'_{cp}], [M'_{pc}], [L'_{pp}]$, and $[R'_{pp}]$ do not depend on the angle θ.

Starting from the known expression for the electromagnetic torque

$$T = \frac{1}{2}[i_t]\frac{d[L]}{d\theta}[i],$$

where

$$[i] = \begin{bmatrix} i_c \\ i_p \end{bmatrix} \quad \text{and} \quad [L] = \begin{bmatrix} L_{cc} & M_{cp} \\ M_{pc} & L_{pp} \end{bmatrix}, \tag{11}$$

after the transformations we obtain

$$T = \frac{1}{2}\left[i'_{ct} \quad i'_{pt}\right] \begin{bmatrix} [A_{ct}^{-1}]\dfrac{d[L_{cc}]}{d\theta}[A_c^{-1}][i'_c] + [A_{ct}^{-1}]\dfrac{d[M_{cp}]}{d\theta}[A_p^{-1}][i'_p] \\ [A_{pt}^{-1}]\dfrac{d[M_{cp}]}{d\theta}[A_c^{-1}][i'_c] \end{bmatrix}. \tag{12}$$

or, finally:

$$T = {}^3/_2\,[i_{cd} i_{cq} i_{c0} i_{pd} i_{pq} i_{p0}] \times \left[\begin{array}{ccccc} 0 & 0.5L_m + M_0 & (M_0 - L_m)\sin 3\theta & 0 & -0 \\ 0.5L_m + M_0 & 0 & (M_0 - L_m)\cos 3\theta & 0.25(M_f + M_g) & \\ (M_0 - L_m)\sin 3\theta & (M_0 - L_m)\cos 3\theta & 0 & 0 & \\ 0 & 0.25(M_f + M_g) & 0 & 0 & \\ -0.5M_h & 0 & 0 & 0 & \\ 0 & 0.25(M_f - M_g) & 0 & 0 & \end{array} \right. \tag{13}$$

Now to the equations obtained above for the voltages and flux linkages of the stator and rotor, (8) and (10), we add the equation of motion of the rotor

$$T_{\mathrm{d}} - T = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}. \tag{14}$$

Here T_{d} is the torque of the prime mover; J is the moment of inertia of the rotor of the synchronous generator, the turbine shaft, and the parts rotating together with them. Thus we have obtained 13 equations, (8), (10), and (14), relating 13 unknowns i_{cd}, i_{cq}, i_{c0}, ψ_{cd}, ψ_{cq}, ψ_{c0}, i_{pd}, i_{pq}, i_{p0}, ψ_{pd}, ψ_{pq}, ψ_{p0}, and θ. Solving the system of equations obtained by one of the numerical methods—and most expediently with the aid of an integrator—makes it possible to calculate any transient electromechanical processes in a synchronous machine.

Equalities (9) and (13) prove that, in the general case, in order to eliminate periodic coefficients from the original equations of a salient-pole synchronous machine it is still not sufficient to rotate the coordinate axes at the rotor speed. For example, when $L_m \neq M_0$, the periodic coefficients ($\sin 3\theta$ and $\cos 3\theta$) are not eliminated from the original equations of the machine, despite the fact that they refer to coordinate axes rigidly connected with its rotor (see (9) and (13)). Under these same conditions ($L_m \neq M_0$), the equations relating the zero components of the currents, voltages, and flux linkages no longer constitute a separate system solved independently of the equations for the direct- and quadrature-axis components, since the direct and quadrature components of the stator flux linkages will depend on the zero components of the stator current, while the zero component of the stator flux linkage will depend on all components of its currents. If L_m differs substantially from M_0, then, in calculating the stator flux linkages and the electromagnetic torque, one must use the equations given above, and not Park' s equations, as was noted by A. G. Iosifyan[6]. Of course, when $L_m = M_0$, Park' s well-known equations are obtained as a special case from the equations presented above.

Moscow Power Engineering Institute
named after V. M. Molotov

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Note: Figure translations are in progress. See original paper for figures.