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Abstract

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ON THE FORCES OF INTERACTION BETWEEN THE EARTH' S CORE AND SHELL ARISING AS A RESULT OF NUTATION

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ASTRONOMY

In considering the rotational motion of the Earth' s shell, it is necessary to take into account both the action of the external perturbing forces of attraction of the Moon and the Sun, and the forces of interaction between the shell and the core. Let the moments of these forces be equal, respectively, to $\mathbf{L}_s$ and $\mathbf{N}$. Then

$$\dot{\mathbf{G}}_s = \mathbf{L}_s + \mathbf{N}, \tag{1}$$

where $\mathbf{G}_s$ is the angular momentum of the shell. Let us denote by $\mathbf{G}$ the angular momentum of the entire Earth. The time derivative of $\mathbf{G}$ is represented, as is known, by a vector lying in the plane of the equator. In this plane we construct coordinate axes with origin at the center of the Earth; direct the axis OX toward the vernal equinox of some initial epoch. Then we shall have:

$$\dot{\mathbf{G}} = g(\sin\theta\,\dot{\psi} + i\dot{\theta}). \tag{2}$$

Here g is the modulus of the vector $\mathbf{G}$; θ is the inclination of the ecliptic to the equator; $\psi = f(t)$, $\theta - \theta_0 = f'(t)$ are the precession and nutation in longitude and obliquity. The functions $f(t)$ and $f'(t)$ are sums of secular and periodic terms. The coefficients of these terms were calculated on the basis of the theory of rotation of an absolutely rigid Earth, but they do not change under any other admissible assumptions about its mechanical properties. In particular, for the principal and semimonthly terms of nutation we have the following theoretical expressions:

$$\begin{aligned} \sin\theta\,\psi &= -6'', 869 \sin\Omega - 0'', 0812 \sin 2\ , \\ \theta - \theta_0 &= +9'', 220 \cos\Omega + 0'', 0834 \cos 2\ , \end{aligned} \tag{3}$$

where Ω is the longitude of the ascending node of the lunar orbit and   is the mean longitude of the Moon. For obvious reasons, we use here that value of the constant of nutation ν which is obtained theoretically on the basis of the known relation between this constant, the mechanical flattening of the Earth H, and the ratio μ of the mass of the Moon to the mass of the Earth:

$$\nu = 231981'', 8H\frac{\mu}{1+\mu}.$$

We adopted $\nu = 9'', 220$, and with this value calculated the other coefficients; moreover, for the second terms of the right-hand sides of equations (3) we took them with a larger number of digits than for the first, since from observations the expression for the semimonthly term of nutation is obtained more accurately.

If there were no interaction between the Earth's core and shell, i.e., if the moment $\mathbf{N}$ in equation (1) could be put equal to zero, then, as is easy to understand, for the time derivative of $\mathbf{G}_s$ we would obtain the expression

$$\dot{\mathbf{G}}_s = hg_s(\sin\theta\,\dot{\psi} + i\dot{\theta}), \tag{4}$$

in which g_s is the modulus of the vector $\mathbf{G}_s$ and h is the ratio of the mechanical flattening of the shell to the mechanical flattening of the whole Earth. Expression (4) may be used to replace L_s in equation (1).

Since the observer is always connected with the Earth' s shell, from astronomical observations we can obtain the right-hand sides of the equations $\psi_r = f_r(t)$, $\theta_r - \theta_0 = f'_r(t)$, which describe the motion precisely of the vector $\mathbf{G}_s$.

It does not seem possible to determine from observations the coefficients of all terms of the nutation, and we have confined ourselves only to the principal and semi-monthly terms. The coefficients of the principal term of the nutation were obtained by us from an analysis of 135 thousand observations at the latitude stations Carloforte, Mizusawa, and Ukiah from 1900 to 1934 ([1]); in determining the coefficients of the semi-monthly term of the nutation ([2]) we also took into account the results of A. Ya. Orlov ([3]) and Morgan ([4]), so that the total number of observations on which the values of these coefficients are based reached 232 thousand. The results obtained by us may be represented as follows:

$$\begin{aligned}\sin\theta\psi_r &= -6''.853\sin\Omega + 0''.008\cos\Omega - 0''.0866\sin 2\zeta + 0''.0019\cos 2\zeta,\\ \theta_r - \theta_0 &= +9''.198\cos\Omega - 0''.001\sin\Omega + 0''.0894\cos 2\zeta + 0''.0019\sin 2\zeta.\end{aligned} \tag{5}$$

We can now determine $\dot{\mathbf{G}}_s$ from observations if we substitute (5) into the formula

$$\dot{\mathbf{G}}_s = g_s(\sin\theta\dot{\psi}_r + i\dot{\theta}_r). \tag{6}$$

Since the vector $\mathbf{N}$ lies in the plane of the equator, it may be represented in the form $X_n + iY_n$ and, bearing in mind (1), written as

$$X_n + iY_n = g_s[\sin\theta(\dot{\psi}_r - h\dot{\psi}) + i(\dot{\theta}_r - h\dot{\theta})]. \tag{7}$$

It is of interest to compare this expression with the one we would have in the case of a complete connection between the core and the shell, i.e., if the core were solid. We shall denote the moment of the forces acting in this case from the core on the shell by $X'_n + iY'_n$.

If motion of the core relative to the shell is impossible, the moment $\mathbf{G}_s$ is practically collinear with $\mathbf{G}$. It follows from this that for substitution in (2) one must now take the theoretical expression for the nutation (3), and we obtain:

$$X'_n + iY'_n = g_s(1-h)(\sin\theta\dot{\psi} + i\dot{\theta}). \tag{8}$$

To compute h, we borrowed the following data from Bullen ([5]):

$$\varepsilon = \frac{C_n - A_n}{A_n} = 0.00260, \qquad \frac{A_n}{A} = 0.112, \tag{9}$$

where C_n and A_n are the principal moments of inertia of the core, and A is the equatorial moment of inertia of the whole Earth. With these data we obtain $h = 1.027$. We substitute (3) into equation (8), and in doing so represent Ω and 2ζ in the form $\Omega = \alpha t$, $2\zeta = \beta t$, where $\alpha = -0.000146\,n$, $\beta = 0.073\,n$, where n is the angular velocity of the Earth' s rotation and t is time. After simple transformations we obtain

$$X'_n + iY'_n = \alpha g_s(0''.217e^{+i\alpha t} - 0''.032e^{-i\alpha t}) + \beta g_s(0''.0023e^{+i\beta t} - 0''.0001e^{-i\beta t}). \tag{10}$$

The moment of the forces arising as a result of the principal nutational motion may, therefore, be represented in the form of the sum of two vectors:

$$\mathbf{U}_1 = 0''.217\alpha g_s e^{+i\alpha t}, \qquad \mathbf{U}_2 = -0''.032\alpha g_s e^{-i\alpha t}. \tag{11}$$

Since α is negative, the first vector rotates in space clockwise, the second in the opposite direction. Relative to

of the Earth itself, both vectors rotate clockwise with angular velocities $-n+\alpha$ and $-n-\alpha$, respectively.

The moment arising as a result of the fortnightly nutation is also represented by two vectors:

$$\mathbf{V}_1 = 0''.0023\beta g_s e^{+i\beta t}, \qquad \mathbf{V}_2 = -0''.0001\beta g_s e^{-i\beta t}. \tag{12}$$

Further, substituting (3) and (5) into equation (7), we have:

$$X_n + iY_n = \alpha g_s \left(0''.236e^{+i\alpha t} - 0''.035e^{-i\alpha t} + 0''.004ie^{+i\alpha t} - 0''.004ie^{-i\alpha t}\right) +$$

$$+\beta g_s \left(-0''.0010e^{+i\beta t} - 0''.0024e^{-i\beta t} + 0''.0019ie^{+i\beta t}\right) =$$

$$= (1.09 + 0.02i)\mathbf{U}_1 + (1.09 + 0.13i)\mathbf{U}_2 + (-0.43 + 0.83i)\mathbf{V}_1 + 24\mathbf{V}_2. \quad (13)$$

It is difficult to give an estimate of the accuracy of the result obtained, but it should be emphasized that we are dealing with such small quantities that they are only barely detectable in the analysis of the longest series of astrometric observations. Nevertheless, the following conclusion, which for the time being is only qualitative in character, apparently deserves confidence.

The influence of the mobility of the core relative to the Earth' s shell has the following consequences:

1. An increase in the modulus of the vector $\mathbf{U}_1$.
2. A reversal of the direction of the vector $\mathbf{V}_1$.
3. A deflection of the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ toward the side opposite to the direction of rotation of these vectors relative to the Earth.

Results 1 and 2 appear at first sight mutually contradictory. However, this contradiction is easily explained if one uses the theory of the rotational motion of the Earth with a liquid core, even in its simplest form, in which this theory was originally developed by F. Sludsky ([6]), and later by A. Poincaré ([7]), and presented in H. Lamb' s course on hydrodynamics ([8]). To calculate the moment of the forces acting from the core on the shell, let us take the following equations of motion (([8]), p.916):

$$A\dot{\vec{\omega}} + F\dot{\vec{\omega}}_n - i(C - A)n\vec{\omega} + iFn\vec{\omega}_n = ke^{i\sigma t}, \quad (14)$$

$$F\dot{\vec{\omega}} + A_n\dot{\vec{\omega}}_n + iC_n\vec{\omega}_n = 0. \quad (15)$$

Here $\vec{\omega} = p + iq$; $\vec{\omega}_n = p_n + iq_n$; p, q are the projections of the angular velocity of the Earth's rotation onto two mutually perpendicular axes lying in the equatorial plane and fixed to the Earth' s shell; p_n, q_n are the projections onto the same axes of the angular velocity of rotation of the core relative to the shell (here by rotation we mean "elliptical rotation" in the sense given to this term by N. E. Zhukovsky ([9])); F is a quantity having the dimension of a moment of inertia and, in the case considered, equal to $A_n\sqrt{1-\varepsilon^2}$.

Let

$$\gamma = \frac{C_n - A_n}{C - A}.$$

Equation (14) can be transformed to the form

$$A_s\dot{\vec{\omega}} - i(C_s - A_s)n\vec{\omega} = (1-\gamma)ke^{i\sigma t} + X_n + iY_n,$$

where C_s, A_s are the principal moments of inertia of the shell. In this case one obtains

$$X_n + iY_n = \gamma(X + iY) - A_n\dot{\vec{\omega}} - F\dot{\vec{\omega}}_n + i(C_n - A_n)n\vec{\omega} - iFn\vec{\omega}_n. \tag{16}$$

In the absence of relative motion of the core, $\omega_n = 0$, and

$$X'_n + iY'_n = \gamma(X + iY) - A_n\dot{\vec{\omega}} + i(C_n - A_n)n\vec{\omega}. \tag{17}$$

Solution of equations (14) and (15)

$$\vec{\omega} = -\frac{A_n\sigma + C_n n}{\Delta(\sigma)}\, ike^{i\sigma t}, \qquad \vec{\omega}_n = \frac{F\sigma}{\Delta(\sigma)}\, ike^{i\sigma t}, \tag{18}$$

where

$$\Delta(\sigma) = \begin{vmatrix} A\sigma - (C-A)n & F(\sigma + n) \\ F\sigma & A_n\sigma + C_n n \end{vmatrix}. \tag{19}$$

After substituting these values of $\vec{\omega}$ and $\vec{\omega}_n$ into equations (16) and (17), and after some transformations, we find

$$\frac{X_n}{X'_n} = \frac{Y_n}{Y'_n} = 1 + \varkappa(\sigma + n), \qquad \varkappa = \frac{F^2\sigma}{S},$$

$$S = \gamma\Delta(\sigma) - (A_n\sigma + C_n n)[A_n\sigma - (C_n - A_n)n], \tag{20}$$

and, substituting numerical values, we obtain

	Principal term of nutation (vector $\mathbf{U}_1$)	Semimonthly term of nutation (vector $\mathbf{V}_1$)
	$\sigma = -n + \alpha = (-1 - 0.000146)n$	$-n + \beta = (-1 + 0.073)n$

	Principal term of nutation (vector $\mathbf{U}_1$)	Semimonthly term of nutation (vector $\mathbf{V}_1$)
	$\Delta(\sigma) = -0.00028n^2C^2$	$-0.00695n^2C^2$
	$S = +0.00001n^2C^2$	$+0.00026n^2C^2$
	$\varkappa n = -1400$	-44
	$1 + \varkappa(\sigma + n) = +1.20$	-2.21

We see that the effect of the mobility of the Earth' s core should indeed lead to an increase in the modulus of the vector $\mathbf{U}_1$ and to a reversal in the direction of the vector $\mathbf{V}_1$. It is natural to suppose that the deviation of the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ toward the daily rotation of the Earth is explained by the influence of friction at the boundary of the core.

Of course, quantitative agreement of the conclusions of the theory with observational data could not have been expected, if only because the Earth model used in constructing the theory is extremely simplified: in it, in particular, the elastic deformations of the shell and the viscosity of the core are not taken into account. Moreover, the influence of the core on the motion of the shell is probably not limited to the action of mechanical forces arising at their boundary; forces of another kind, for example magnetic forces, may also play a large role.

For further refinement of the results obtained, it is necessary to analyze new observational data. A significant step in this direction can be made when the observations of the International Latitude Service from 1935 to 1954 are published.

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Note: Figure translations are in progress. See original paper for figures.