ON THE INTRODUCTION OF RADIATION INTO PROBLEMS OF GAS DYNAMICS

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Abstract

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In most works, when solving gas-dynamic problems, the influence of radiation was not taken into account. In doing so, the authors referred either to the fact that, at the existing density, the temperature is too low, or to the exceptional complication of the equations of motion. It will be shown below that neglecting the influence of radiation often leads to large inaccuracies.

The purpose of the present work is to set forth a very simple method that makes it possible to take into account the influence of radiation without any change in the equations of adiabatic motions.

As is known, radiation in thermodynamic equilibrium may be regarded as an ideal gas with adiabatic exponent $\varkappa = {}^4/_3$. The thermodynamic equations of an ideal gas with radiation in equilibrium with it have the form:

$$E = \frac{1}{\varkappa - 1}\frac{p_\mathrm{m}}{\rho} + \frac{1}{{}^4/_3 - 1}\frac{p_\mathrm{i}}{\rho_\mathrm{i}}, \tag{1a}$$

$$S = \frac{R}{\varkappa - 1}\ln\frac{T}{\rho^{\varkappa-1}} + \frac{4a}{3}\left(\frac{T}{\rho_\mathrm{i}^{1/3}}\right)^3; \tag{1 }$$

$$p_\mathrm{m} = \rho R T, \qquad p_\mathrm{i} = \frac{a}{3}T^4; \tag{1 }$$

$$p = p_\mathrm{m} + p_\mathrm{i}. \tag{1 }$$

Here R is the gas constant; a is the radiation-density constant; p_m is the molecular pressure; p_i is the radiation pressure; $\varkappa$ is the adiabatic exponent of the gas; E is the internal energy per unit mass; S is the entropy per unit mass.

These equations are greatly simplified in two cases:

1) $\varkappa = {}^4/_3$;

2) the radiation pressure can be approximated by the formula

$$p_{\mathrm{i}} = (\varkappa - 1)\,\bar{a}T^{\frac{\varkappa}{\varkappa-1}}, \tag{2}$$

i.e., when the adiabatic exponents for the gas and for the radiation are equal to each other. In each of these cases one may write:

$$E = \frac{1}{\varkappa - 1}\frac{p}{\rho}, \tag{3a}$$

$$S = f\left[\frac{p}{\rho^{\varkappa}}\right], \tag{3 }$$

$$p = \rho RT + (\varkappa - 1)\bar{a}T^{\frac{\varkappa}{\varkappa-1}}, \tag{3 }$$

since the pressure can be represented in the form

$$\frac{p}{R\rho^{\varkappa}} = \frac{T}{\rho^{\varkappa-1}}\left[1 + (\varkappa - 1)\frac{\bar{a}}{R}\left(\frac{T}{\rho^{\varkappa-1}}\right)^{\frac{1}{\varkappa-1}}\right]. \tag{4}$$

The form of the function $f\left[\frac{p}{\rho^{\chi}}\right]$ can be obtained by solving the last equation, but it is immaterial for what follows.

From formulas (3) it is clear that, under assumptions 1) or 2), the equations of adiabatic motion of an ideal gas and the boundary conditions for them do not change when radiation is introduced. Thus, the solutions p, ρ, and v (velocity) have the same form as in the case when radiation is neglected. The difference appears in the values of the temperatures; but it is precisely the temperature values that are important for various applications, and they must be calculated by formula (3). In this case the problem without taking radiation into account is self-similar, whereas the temperature distribution is not self-similar at all because of the appearance of the new dimensional constant $\bar{a}$.

For $\chi = 4/3$, radiation has no effect at all on the mechanical parameters. Therefore one may hope to obtain a fairly good approximate solution even for $\chi \neq 4/3$, and also in the case when equation (2) is not satisfied with sufficient accuracy, while using the equations of state (1) and (1).

The assertion made in the introduction is thereby proved.

Let us apply these general considerations to a very interesting and topical case in which exact solutions of the gas-dynamic equations are known, namely to the problem of a strong explosion.

Fig. 1

Fig. 1

Figure 1: Fig. 1

This problem was solved by L. I. Sedov (^{1}) under certain simplifications. Namely, he assumed that: 1) the explosion is sufficiently strong, so that the energy of the undisturbed gas may be neglected; 2) the transfer of energy is purely macroscopic; 3) light pressure is inessential, so that the gas, being ideal, obeys the Clapeyron equation. In this formulation the problem is self-similar, and L. I. Sedov succeeded in finding its solution in analytic form.

Because of the high temperatures that develop in strong explosions, however, light pressure must play a large role in the initial stage of propagation of the shock wave. Moreover, as is evident from L. I. Sedov' s solutions, the density becomes very small inside the sphere whose radius is approximately equal to half the radius of the shock wave. Application of the Clapeyron equation to such a rarefied gas must lead to large inaccuracies. In particular, this leads to the result that in the central zone the temperature is obtained as always very high compared with the temperature behind the wave, and at the center even infinite.

It follows from the preceding that the solutions for p, ρ, and v found by L. I. Sedov remain valid also in the presence of radiation. Below it is shown how the temperature distribution changes.

Instead of the temperature, we introduce the dimensionless variable according to the formula

$$T = \frac{p}{R\rho}\,\theta. \tag{5}$$

The equation of state (1) takes the form

$$1 = \theta + \frac{a}{3R^4}\frac{p^3}{\rho^4}\theta^4. \tag{6}$$

The solution of this algebraic equation

$$\theta = \theta\left[\frac{3R^4}{a}\frac{\rho^4}{p^3}\right] \tag{7}$$

is presented in Fig. 1. As can be seen, the temperature distribution is determined by the additional dimensional constant R^4/a and is not self-similar.

As two dimensionless variables on which θ depends, we take the variable λ, introduced by L. I. Sedov:

Fig. 2

Figure 2: Fig. 2

$$\lambda = \frac{E}{\rho_1} \frac{t^2}{r^{\nu+2}} \tag{8}$$

and the variable

$$\mu = \frac{3R^4 \rho_1}{a} \frac{t^6}{r^6}. \tag{9}$$

Equation (7) can be written in the form

$$\theta = \theta \left[\frac{(\chi+1)^7 (\nu+2)^6}{2^9 (\chi-1)^4} \mu^* \frac{(\rho/\rho^*)^4}{(p/p^*)^3} \right], \tag{10}$$

Here $\nu = 3$ for a spherical wave, $\nu = 2$ for a cylindrical wave, and $\nu = 1$ for a plane wave; the asterisk indicates that the value of the given quantity is taken at the shock wave.

Fig. 2

One can form the ratio

$$\frac{T}{T^*} = \frac{\left(\dfrac{p}{p^*}\right) \theta \left[\dfrac{(\chi+1)^7 (\nu+2)^6}{2^9 (\chi-1)^4} \mu^* \dfrac{\left(\dfrac{\rho}{\rho^*}\right)^4}{\left(\dfrac{p}{p^*}\right)^3} \right]}{\theta \left[\dfrac{(\chi+1)^7 (\nu+2)^6}{2^9 (\chi-1)^4} \mu^* \right]}. \tag{11}$$

In Fig. 2, the curves determined by formula (11) are compared, for $\nu = 3$ and $\chi = 1.4$, with the curve from the book by L. I. Sedov.^2 In the calculations, the formulas and tables from that book were used. As might have been expected, the curves (11) approach the curve of L. I. Sedov as $\mu^* \to \infty$, i.e., after a fairly long time interval following the explosion. At the center the temperature is always finite and decreases with time according to the law

$$T(0,t) = k_3 \left[\frac{3(\chi+1)^7 (\nu+2)^6}{2^9} \frac{\rho_1}{a} \right]^{1/4} \left(\frac{E}{\rho_1} \right)^{1/2(\nu+2)} t^{-\nu/2(\nu+2)} \tag{12}$$

(k_3 is taken from ^2).

Let us also note that, at the initial stage, the instantaneous ratio T/T^* is found to be almost constant with respect to the radius, so that, apparently, thermal conductivity does not play a major role. In this connection let us recall the recently published work,^3 where the problem of an explosion is solved for a gas obeying the Clapeyron equation and having infinite thermal conductivity in the disturbed zone.

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Note: Figure translations are in progress. See original paper for figures.