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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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ON THE TRANSFORMATION OF THE EQUATIONS OF TRANSIENT PROCESSES OF AN r, L, C CIRCUIT TO A ROTATING COORDINATE SYSTEM

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When calculating transient electromechanical processes in an electrical system containing, in addition to rotating machines (synchronous and asynchronous), also a number of static (non-rotating) elements (transmission lines, loads, compensating reactors, series capacitances, etc.), it is inadvisable to use equations for all these elements written in phase coordinates. They are not only more complicated, but also less transparent from the point of view of the physics of the processes described. Accordingly, the equations of all synchronous machines are referred to coordinate axes rigidly connected with their rotors, which is necessary in order to eliminate periodic coefficients from their original equations written in phase coordinates ([1,2]). Consequently, in the circuit of Fig. 1, the currents i_1, i_2 and voltages u_1, u_2 of the first and second synchronous generators ($SG1$ and $SG2$) must be referred to coordinate axes rigidly connected with their own rotors. In exactly the same way, the equations of all static elements of the circuit should be referred to a rotating coordinate system. The latter must be chosen in the most rational way, namely so that the set of nonlinear equations of the entire circuit as a whole would be as simple as possible ([1,2]).

Fig. 1

In view of what has been said, it appears necessary to derive the equations of transient processes in a compensated line connecting two generator stations $SG1$ and $SG2$ (Fig. 1); moreover, this line, in other words the series circuit r, L, C, may be regarded as a general case of a static element of the system. We refer these equations to coordinate axes rigidly connected with the third station and rotating, generally speaking, with an arbitrary angular velocity ω_k.

We have the equations of the circuit section 1—3 (Fig. 2) in phase coordinates

Fig. 2

Figure 2: Fig. 2

$$[u_3] = [u_1] - \frac{r_\ell}{2}[i_\ell] - \frac{[L_\ell]}{2}\frac{d[i_\ell]}{dt}, \tag{1}$$

where

$$[u_3] = \begin{bmatrix} u_{3a} \\ u_{3b} \\ u_{3c} \end{bmatrix}; \qquad [i_\ell] = \begin{bmatrix} i_{\ell a} \\ i_{\ell b} \\ i_{\ell c} \end{bmatrix}; \qquad [L_\ell] = \begin{bmatrix} L_\ell & M_\ell & M_\ell \\ M_\ell & L_\ell & M_\ell \\ M_\ell & M_\ell & L_\ell \end{bmatrix}; \qquad [u_1] = \begin{bmatrix} u_{1a} \\ u_{1b} \\ u_{1c} \end{bmatrix}. \tag{2}$$

As the transformation matrix we choose Park' s matrix $[A_k]$ ([3])

$$[A_k] = \frac{2}{3}\begin{bmatrix} \cos\theta_k & \cos(\theta_k - 120^\circ) & \cos(\theta_k + 120^\circ) \\ -\sin\theta_k & -\sin(\theta_k - 120^\circ) & -\sin(\theta_k + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \tag{3}$$

where

$$\theta_k = \int_0^t \omega_k \, dt + \theta_{k0}. \tag{4}$$

Fig. 2

The voltage, current, and inductance matrices after transformation will be denoted by a prime index:

$$[u_3'] = \begin{bmatrix} u_{3d} \\ u_{3q} \\ u_{30} \end{bmatrix} = [A_k][u_3]; \qquad [i_\ell'] = \begin{bmatrix} i_{\ell d} \\ i_{\ell q} \\ i_{\ell 0} \end{bmatrix} = [A_k][i_\ell], \tag{5}$$

$$[u_1'] = \begin{bmatrix} u_{1d} \\ u_{1q} \\ u_{10} \end{bmatrix} = [A_1][u_1]; \qquad [L_\ell'] = [A_k][L_\ell][A_k^{-1}] = \begin{bmatrix} L_{\ell 1} & 0 & 0 \\ 0 & L_{\ell 1} & 0 \\ 0 & 0 & L_{\ell 0} \end{bmatrix}, \tag{6}$$

where $[A_1]$ and θ_1, $[A_2]$ and θ_2 are obtained from (3) and (4), respectively, for $k = 1$ and $k = 2$ (Fig. 3); $L_{\ell 1} = L_\ell - M_\ell$, $L_{\ell 0} = L_\ell + 2M_\ell$ are the inductances of the positive- and zero-sequence lines; the inverse matrix $[A_k^{-1}]$ is

Fig. 3

Figure 3: Fig. 3

$$[A_k^{-1}] = \begin{bmatrix} \cos\theta_k & -\sin\theta_k & 1 \\ \cos(\theta_k - 120^\circ) & -\sin(\theta_k - 120^\circ) & 1 \\ \cos(\theta_k + 120^\circ) & -\sin(\theta_k + 120^\circ) & 1 \end{bmatrix} \tag{7}$$

and, moreover,

$$[u_1] = [A_1^{-1}][u_1']; \qquad [u_2] = [A_2^{-1}][u_2']; \qquad [i_\ell] = [A_k^{-1}][i_\ell']. \tag{8}$$

Multiplying both sides of (1) on the left by $[A_k]$, taking (2)-(7) into account, after transformation we obtain

$$[u_3'] = [A_k][A_1^{-1}][u_1'] - \frac{r_\ell}{2}[i_\ell'] - \frac{[L_\ell']}{2}\frac{d[i_\ell']}{dt} + \frac{1}{2}\frac{d[A_k]}{dt}[A_k^{-1}][L_\ell'][i_\ell']. \tag{9}$$

Carrying out the differentiation $\frac{d[A_k]}{dt}$ and multiplying the matrices $[A_k][A_1^{-1}]$ and $\frac{d[A_k]}{dt}[A_k^{-1}][L_\ell']$, we obtain

$$[u_3'] = \begin{bmatrix} \cos(\theta_k - \theta_1) & \sin(\theta_k - \theta_1) & 0 \\ -\sin(\theta_k - \theta_1) & \cos(\theta_k - \theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}[u_1'] - \frac{r_\ell}{2}[i_\ell'] - \frac{[L_\ell']}{2}\frac{d[i_\ell']}{dt} + \frac{1}{2}\begin{bmatrix} 0 & L_{\ell 1} & 0 \\ -L_{\ell 1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}[i_\ell']\frac{d\theta_k}{dt}. \tag{10}$$

Now we can immediately write, for the section of the circuit 4—2,

$$\begin{bmatrix} \cos(\theta_k - \theta_2) & \sin(\theta_k - \theta_2) & 0 \\ -\sin(\theta_k - \theta_2) & \cos(\theta_k - \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}[u_2'] =$$

$$= [u_4'] - \frac{r_\ell}{2}[i_\ell'] - \frac{[L_\ell']}{2}\frac{d[i_\ell']}{dt} + \frac{1}{2}\begin{bmatrix} 0 & L_{\ell 1} & 0 \\ -L_{\ell 1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}[i_\ell']\frac{d\theta_k}{dt}. \tag{11}$$

Fig. 3

Further, for the longitudinal capacitance C_k, we have the following initial equations of Ohm' s law for the instantaneous values of phase currents and voltages:

$$[u_3] - [u_4] = \frac{1}{C_k}\int [i_\ell]\,dt. \tag{12}$$

Multiplying both sides of (12) on the left by $[A_k]$. Taking into account the equality

$$[u_4'] = \begin{bmatrix} u_{4d} \\ u_{4q} \\ u_{40} \end{bmatrix} = [A_k][u_4] = [A_k] \begin{bmatrix} u_{4a} \\ u_{4b} \\ u_{4c} \end{bmatrix}, \tag{13}$$

and also, on the basis of (5), after transformations we obtain

$$[u_3'] - [u_4'] = \frac{1}{C_k}[A_k] \int [i_\ell]\, dt = \frac{1}{C_k} \int [i_\ell']\, dt + \int \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ([u_3'] - [u_4']) \frac{d\theta_k}{dt}\, dt. \tag{14}$$

Substituting $[u_3']$ and $[u_4']$ from (10) and (11) into (14), we obtain, in the most general case, the equation of the longitudinal branch r_ℓ, L_ℓ, C_k (i.e., in other words, the circuit r, L, C), referred to a coordinate system rotating with arbitrary angular velocity ω_k:

$$\begin{aligned}
&\begin{bmatrix} \cos(\theta_k - \theta_1) & \sin(\theta_k - \theta_1) & 0 \\ -\sin(\theta_k - \theta_1) & \cos(\theta_k - \theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_1'] - \begin{bmatrix} \cos(\theta_k - \theta_2) & \sin(\theta_k - \theta_2) & 0 \\ -\sin(\theta_k - \theta_2) & \cos(\theta_k - \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_2'] \\
&\quad - r_\ell [i_\ell'] - [L_\ell'] \frac{d[i_\ell']}{dt} + L_{\ell 1} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} [i_\ell'] \frac{d\theta_k}{dt} \\
&= \frac{1}{C_k} \int [i_\ell']\, dt + \int \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left\{ \begin{bmatrix} \cos(\theta_k - \theta_1) & \sin(\theta_k - \theta_1) & 0 \\ -\sin(\theta_k - \theta_1) & \cos(\theta_k - \theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_1'] \right. \\
&\quad - \begin{bmatrix} \cos(\theta_k - \theta_2) & \sin(\theta_k - \theta_2) & 0 \\ -\sin(\theta_k - \theta_2) & \cos(\theta_k - \theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_2'] - r_\ell [i_\ell'] - [L_\ell'] \frac{d[i_\ell']}{dt} \\
&\quad \left. + L_{\ell 1} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} [i_\ell'] \frac{d\theta_k}{dt} \right\} \frac{d\theta_k}{dt}\, dt.
\end{aligned} \tag{15}$$

From this general equation one can obtain a number of equations for practically interesting special cases:

1. The equation of an uncompensated line r_ℓ, L_ℓ (a longitudinal branch r, L) is obtained from (10), (11), and (14), putting in (14) $C_k = \infty$ and, consequently, in (11) $[u_3'] = [u_4']$:

$$\begin{bmatrix} \cos(\theta_k-\theta_2) & \sin(\theta_k-\theta_2) & 0 \\ -\sin(\theta_k-\theta_2) & \cos(\theta_k-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_2'] = \begin{bmatrix} \cos(\theta_k-\theta_1) & \sin(\theta_k-\theta_1) & 0 \\ -\sin(\theta_k-\theta_1) & \cos(\theta_k-\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_1']$$
$$- r_\ell [i_\ell'] - [L_\ell'] \frac{d[i_\ell']}{dt} + L_{\ell 1} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} [i_\ell'] \frac{d\theta_k}{dt}. \tag{16}$$

2. The equation for the longitudinal capacitance C_k is obtained from (15) for $r_\ell = 0$, $L_{\ell 1} = 0$, and $L_{\ell 0} = 0$:

$$\begin{bmatrix} \cos(\theta_k-\theta_1) & \sin(\theta_k-\theta_1) & 0 \\ -\sin(\theta_k-\theta_1) & \cos(\theta_k-\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_1'] - \begin{bmatrix} \cos(\theta_k-\theta_2) & \sin(\theta_k-\theta_2) & 0 \\ -\sin(\theta_k-\theta_2) & \cos(\theta_k-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_2']$$
$$= \frac{1}{C_k} \int [i_\ell']\, dt + \int \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \left\{ \begin{bmatrix} \cos(\theta_k-\theta_1) & \sin(\theta_k-\theta_1) & 0 \\ -\sin(\theta_k-\theta_1) & \cos(\theta_k-\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_1'] \right.$$
$$\left. - \begin{bmatrix} \cos(\theta_k-\theta_2) & \sin(\theta_k-\theta_2) & 0 \\ -\sin(\theta_k-\theta_2) & \cos(\theta_k-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} [u_2'] \right\} \frac{d\theta_k}{dt}\, dt. \tag{17}$$

The equations for a transverse branch r_ℓ, L_ℓ, C_k, for a static load or a compensating reactor (in other words, a transverse branch r_ℓ, L_ℓ), and for a transverse capacitance C_k are obtained by putting $[u_2'] = 0$ in equations (15), (16), and (17).

The formulas (15)-(17) obtained in the article are needed when preparing the equations of the indicated static elements forming part of an electrical system for which electromechanical transients are to be calculated. They should be used directly in preparing the equations of the entire system as a whole–in the simplest form, if the equations are to be solved on an integrator. They should also be used in an analytical solution of the problem, for example, by means of any of the numerical methods for solving a system of nonlinear integro-differential equations.

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Note: Figure translations are in progress. See original paper for figures.