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Abstract

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OSCILLATIONS OF AN INFINITE GAS CYLINDER WITH SELF-GRAVITY IN A MAGNETIC FIELD

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Let us consider one-dimensional unsteady motions of a gas, which may be associated with problems of the motion of cosmic masses under the action of magnetic fields. As the basic assumption we shall take the electrical conductivity of the gas to be so large that the magnetic lines of force may practically be regarded as "frozen" into the medium. Below we consider problems of radial motions of a gas with cylindrical symmetry under the action of Newtonian gravitational forces and an internal magnetic field. We investigate gas motions in which the velocity depends linearly on the distance from the axis of symmetry. Motions of this type were studied earlier in the works of L. I. Sedov ([1,2]), M. S. Lidov ([3]), and A. G. Kulikovskii ([4]).

1. Suppose that the internal magnetic field is directed along the axis of symmetry. Such a problem on radial oscillations of small amplitude in the linear formulation was considered by Chandrasekhar and Fermi ([5]) in connection with the study of problems of gravitational stability.

In this case the system of equations of motion in cylindrical coordinates has the form ([4])

$$\frac{\partial^2 r}{\partial t^2} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{2Gm}{r} - \frac{1}{8\pi\rho}\frac{\partial H^2}{\partial r};$$

$$\rho = \rho_0(r_0)\frac{r_0}{r}\frac{\partial r_0}{\partial r}; \qquad p = p_0(r_0)\frac{\rho^\gamma}{\rho_0^\gamma};$$

$$m = m_0 = 2\pi\int_0^{r_0} \rho_0(r_0)\, r_0\, dr_0; \qquad H = H_0(r_0)\frac{r_0}{r}\frac{\partial r_0}{\partial r}, \tag{1}$$

where t is time; r_0 is the Lagrangian coordinate; the remaining notation is clear from the equations.

Let us study an exact particular solution of system (1), having the form:

$$r = r_0\mu(t); \qquad v = r_0\mu'(t) = r\frac{\mu'(t)}{\mu(t)};$$

Fig. 1. Form of the function $f(\mu)$ and the law of motion of a gas particle in cases I and II

Figure 1: Fig. 1. Form of the function $f(\mu)$ and the law of motion of a gas particle in cases I and II

$$\rho = \rho_0 \mu^{-2}(t); \qquad p = p_0(r_0)\mu^{-2\gamma}(t); \qquad H^2 = H_0^2(r_0)\mu^{-4}(t),$$

where the function $\mu(t)$ satisfies the equation

$$t = \pm \int \frac{d\mu}{\sqrt{f(\mu)}},$$

where

$$f(\mu) = \frac{A}{\gamma - 1}\frac{1}{\mu^{2(\gamma-1)}} + \frac{B}{\mu^2} - 2C\ln\mu + D;$$

$$A = \frac{1}{\rho_0 r_0}\frac{\partial p_0}{\partial r_0}; \qquad B = \frac{1}{8\pi}\frac{1}{\rho_0 r_0}\frac{\partial H_0^2}{\partial r_0}; \qquad C = \frac{2Gm_0}{r_0^2},$$

whence it follows immediately that

$$\rho_0 = \text{const}; \qquad p_0 = \frac{A\rho_0 r_0^2}{2} + \bar{p}; \qquad H_0^2 = 4\pi B\rho_0 r_0^2 + \bar{H}^2. \tag{*}$$

It is obvious that always $C > 0$. If the forces of gravity are not taken into account, then $C = 0$. Depending on the different values of the quantities A, B, C, and D, the motion of the gas will be different.

Let us note all possible cases:

I. The gradients of pressure and of magnetic-field intensity at the initial moment are positive ($A > 0;\ B > 0$).

The function $f(\mu)$ has a root $\mu_1 > 1$; $f'(\mu) < 0$.

a) If $(dr/dt)_{t=0} < 0$, then the gas contracts to a point in a finite interval of time.

b) For $(dr/dt)_{t=0} > 0$, the gas expands to a certain maximum volume; in doing so its velocity becomes zero and, under the action of the forces, the reverse motion of the gas toward the center begins (Fig. 1 I). One may imagine that, with increasing time, the gas again begins to expand and the process repeats, i.e. periodic pulsations of the gas take place.

Fig. 1. Form of the function $f(\mu)$ and the law of motion of a gas particle in cases I and II

II. The gradient of the magnetic-field intensity is negative ($A \neq 0;\ B < 0;\ \gamma < 2$). For $\gamma > 2$ the determining factor is the sign of A.

The $f(\mu)$ have roots $\mu_1 < 1$ and $\mu_2 > 1$. The motion of the gas is periodic oscillations. The period of the oscillations depends on the values of A, B, C, and D. In the particular case $f(\mu)$ may have a double root $\mu_1 = \mu_2 = 1$. This means that at the initial moment no forces act on the gas ($A + B + C = 0$) and the initial velocity is equal to zero

$$\left(\frac{A}{\gamma - 1} + B + D = 0\right).$$

As is seen from Fig. 1 II, this is a position of stable equilibrium.

III. The pressure gradient is negative ($A < 0$), the gradient of the magnetic-field intensity is positive ($B > 0$).

If $f(\mu)$ has a root $\mu < 1$, then the motion is analogous to case II; in the remaining cases it is analogous to case I.

IV. The magnetic field is constant ($B = 0$).

Then for $A > 0$ ($dp_0/dr_0 > 0$) the motion is analogous to case I; for $A < 0$ ($dp_0/dr_0 < 0$) it is analogous to case II.

V. The gas pressure at the initial moment is constant.

The case is completely analogous to the preceding one, with A replaced by B.

VI. The magnetic field is constant, and the forces of gravity are absent.

This case coincides with the case of absence of mass forces (1).

VII. The forces of gravity are absent ($C = 0$).

In this case one of the boundary conditions, $\rho_0 = \text{const}$, drops out; the solution has a more general form and depends on an arbitrary function $\rho_0(r_0)$.

a) For $A \neq 0;\ B > 0;\ D > 0$, depending on the sign of

$$\left|\frac{dr}{dt}\right|_{t=0}$$

and on the location of the roots of $f(\mu)$, there is either contraction of the gas to a point or expansion.

b) For $A \neq 0,\ B > 0;\ D < 0$, compression of the gas in a finite interval of time.

c) $A < 0;\ B < 0;\ D > 0$—dispersal of the gas with finite velocity at infinity.

d) $A > 0;\ B < 0;\ D < 0$—the motion represents periodic oscillations.

e) $A > 0;\ B < 0;\ D > 0$—complete dispersal.

VIII. The pressure and the magnetic-field intensity are constant ($A = 0;\ B = 0$).

The motion of the gas is analogous to case I.

On the basis of the preceding investigation we see that, in the presence of gravitational forces and of a magnetic field directed along the axis of symmetry, and for initial distributions of the type ($*$), dispersal of the gaseous configuration is impossible.

2. Let us dwell further on the case when the magnetic lines of force are closed concentric circles.

The equations of motion in this case differ from the equations given in the work of A. G. Kulikovskii [4] only by the presence of a term taking account of gravitational forces. But the solution of the system of the required type will have a more particular form, namely:

$$r = r_0\mu(t); \qquad \rho = \rho_0\mu^{-2}(t);$$

$$p = p_0(r_0)\mu^{-2\gamma}(t); \qquad H^2 = H_0^2(r_0)\mu^{-2}(t),$$

where

$$\rho_0 = \text{const}; \qquad p_0 = \frac{A\rho_0^2 r_0^2}{r} + \bar{p}; \qquad H_0^2 = 2\pi B\rho_0 r_0^2 + \frac{\overline{H}^2}{r_0^2};$$

$$t = \pm\int\frac{d\mu}{\sqrt{f(\mu)}}; \qquad C = 2\pi G\rho_0;$$

$$f(\mu) = \frac{A}{\gamma-1}\frac{1}{\mu^{2(\gamma-1)}} - 2B\ln\mu - 2C\ln\mu + D, \tag{**}$$

i.e. the solution will not depend on an arbitrary function. The second and third terms in equation ($**$) can be combined by replacing B by $B' = B + C$.

Thus, for motions of the type under consideration, the action of gravitational forces is equivalent to a change in the gradient of the magnetic-field intensity. All the investigations carried out in [4] remain valid with the replacement of B by B'. In this case, for $B' < 0$ and in the presence of gravitational forces, complete dispersal of the gaseous configuration proves possible.

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CITED LITERATURE

[1] L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*, Moscow, 1954.
[2] L. I. Sedov, DAN, 90, No. 5 (1953).
[3] M. L. Lidov, DAN, 97, No. 3 (1954).
[4] A. G. Kulikovskii, DAN, 114, No. 5 (1957).
[5] S. Chandrasekhar, E. Fermi, *Problems of Modern Physics*, No. 2, 108 (1954).

Note: Figure translations are in progress. See original paper for figures.