HYDROMECHANICS

Yu. S. ZAVYALOV

1957

SovietRxiv

Abstract

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Yu. S. ZAVYALOV

ON A CLASS OF PLANE-PARALLEL STEADY VORTICAL MOTIONS OF A GAS

(Presented by Academician L. I. Sedov on 16 IV 1957)

In the monograph of L. I. Sedov (1) and in the work of Yu. V. Rudnev (2) it is shown that, if the velocity v of a plane-parallel steady vortical flow of an ideal compressible fluid has the form $v(p, \psi) = f(p)F(\psi)$, where p is the pressure and ψ is the stream function, then the problem of the vortical motion of a gas reduces to the problem of a potential motion with the same system of streamlines and with velocity $V(p) = f(p)$. In the present note this question is considered for a more general case.

Let the quantity $(1/v)(\partial^2 v/\partial p^2)$ be independent of ψ. Then the function $v(p, \psi)$ must have the form (1)

$$v(p, \psi) = f_1(p)F_1(\psi) + f_2(p)F_2(\psi), \tag{1}$$

where the functions $f_1(p)$ and $f_2(p)$ satisfy the relation

$$\frac{f_1''(p)}{f_1(p)} = \frac{f_2''(p)}{f_2(p)}. \tag{2}$$

Let us consider the equations of plane steady vortical motion of a gas, derived by L. I. Sedov (1), in the form of Yu. V. Rudnev (2):

$$v\frac{\partial^2 \varphi}{\partial p^2} - \frac{\partial^2 v}{\partial p^2}\frac{\partial^2 \varphi}{\partial \theta^2} + 2\frac{\partial v}{\partial p}\frac{\partial \psi}{\partial p} + \frac{\partial v}{\partial \psi}\left(\frac{\partial \psi}{\partial p}\right)^2 - \frac{\partial^2 v}{\partial p^2 \partial \psi}\left(\frac{\partial \psi}{\partial \theta}\right)^2 = 0; \tag{3}$$

$$dz = -\rho_0 e^{i\theta}\left[\left(v\frac{\partial \psi}{\partial p} + i\frac{\partial v}{\partial p}\frac{\partial \psi}{\partial \theta}\right)d\theta + \left(\frac{\partial^2 v}{\partial p^2}\frac{\partial \psi}{\partial \theta} + i\frac{\partial v}{\partial p}\frac{\partial \psi}{\partial p}\right)dp\right], \tag{4}$$

where $z = x + iy$; θ is the angle of inclination of the velocity vector to the x-axis; ρ_0 is a certain constant.

Let us take some function $V(p)$ satisfying the condition

$$\frac{V''(p)}{V(p)} = \frac{1}{v}\frac{\partial^2 v}{\partial p^2}, \tag{5}$$

for example $V(p) = f_1(p)$, and in equations (3), (4) pass to a new variable ψ^* by the formula

$$V(p)(\psi^* - \psi_0^*) = \int_{\psi_0}^{\psi} v(p,\psi)\, d\psi, \tag{6}$$

where ψ_1^* and ψ_0^* are constants.

Then the quasilinear equation (3) is brought to the linear form

$$V(p)\frac{\partial^2 \psi^*}{\partial p^2} - V''(p)\frac{\partial^2 \psi^*}{\partial \theta^2} + 2V'(p)\frac{\partial \psi^*}{\partial p} = 0. \tag{7}$$

Equation (7) coincides with the equation for determining the stream function ψ^* of a potential motion of a gas with velocity $V(p)$.

Formula (4) can then be represented in the form

$$dz = dz^* + dz_1, \tag{8}$$

where dz^* denotes relation (4), written for the aforementioned potential flow with the functions ψ^* and $V(p)$, and

$$z_1 = -i\rho_0 e^{i\theta}\frac{1}{V}U(\psi), \qquad U(\psi) = \int_{\psi_0}^{\psi}\left(V\frac{\partial v}{\partial p} - V'v\right) d\psi. \tag{9}$$

U is a function only of ψ, since $\partial U/\partial p = 0$ by virtue of condition (5).

From formula (6) it is clear that, in contrast to the case considered in ([1],[2]), here the systems of streamlines of the vortical and potential motions are different. Only the initial lines $\psi = \psi_0$ and $\psi^* = \psi_0^*$ will coincide. Moreover, choosing

$$V(\rho) = (\psi_1^* - \psi_0^*)^{-1}\int_{\psi_0}^{\psi_1} v(p,\psi)\, d\psi$$

(condition (5) will then be satisfied), we can make the streamline $\psi = \psi_1 = \text{const}$ coincide with the line $\psi^* = \psi_1^* = \text{const}$. In this case one may take $\psi_0^* = \psi_0$ and $\psi_1^* = \psi_1$.

Obviously, if $v(p,\psi) = f(p)F(\psi)$ and $V(p) = f(p)$, then $U(\psi) = 0$, and formulas (6)–(8) pass into the analogous formulas of work ([2]).

Thus, if the function $v(p, \psi)$ is expressed by formula (1) under condition (2), then the problem of the vortical motion of a gas reduces to the problem of potential motion.

Conversely, to every potential motion with functions ψ^* and $V(p)$ there corresponds a vortical motion with functions φ and $v(p, \psi)$, determined by formulas (5), (6), and (8). Having, for example, a solution for the generalized Prandtl–Meyer potential flow, one can easily construct the solution, previously found by L. I. Sedov ([1]), for the corresponding vortical motion. It is not difficult to find other solutions of the vortical problem as well, including those obtained by us by another method ([3]).

In the work of I. Z. Kalishevich ([4]) it is shown that if, in adiabatic motion of a gas, the total energy is constant and the function characterizing the entropy changes little, then the velocity function can be approximately represented in the form (1). Then, using the results obtained above, we find approximate solutions of the vortical problem. One such solution for the case of supersonic motion is given in ([3]).

Tomsk State University
named after V. V. Kuibyshev

Received
12 IV 1957

CITED LITERATURE

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[3] Yu. S. Zav' yalov, Abstract of dissertation, Tomsk, 1956. [4] I. Z. Kalishevich, DAN, 99, No. 1, 37 (1954).

Note: Figure translations are in progress. See original paper for figures.