PHYSICS

1957

SovietRxiv

Abstract

Full Text

PHYSICS

G. F. FILIPPOV

FIRST EXCITED LEVELS OF AXIALLY SYMMETRIC EVEN-EVEN NUCLEI

(Presented by Academician I. E. Tamm on 13 III 1957)

According to the generalized model ([1]), the Schrödinger equation for heavy nuclei has the form

$$(\hat{H}_s + \hat{H}_p + \hat{H}_{int}) = E\psi, \tag{1}$$

where $\hat{H}_s$ is the Hamiltonian operator of the core; $\hat{H}_p$ is the Hamiltonian operator of the extra-shell nucleons; $\hat{H}_{int}$ is the operator of the interaction energy between the core and the "excess" nucleons. The motion of the core is characterized by three Euler angles and the variables β and γ, where β takes into account the change in the surface area of the nucleus when the shape of the nucleus deviates from spherical, while γ indicates the degree of violation of axial symmetry.

Equation (1) has been solved in the approximation of strong and weak coupling ([2–4]). In the present work an attempt is made to consider, in the strong-coupling approximation, the rotational-vibrational spectrum of axially symmetric nuclei. The assumption of the preservation of axial symmetry is apparently valid for the first excited levels.

The generalized model gives the following Hamiltonian function for free vibrations of the nuclear core ([1]):

$$H' = \frac{1}{2}B(\dot{\beta}^2 + \beta^2\dot{\gamma}^2) + \frac{1}{2}\sum_{\chi=1}^{3}\frac{M_\chi^2}{T_\chi} + \frac{1}{2}C\beta^2,$$

where

$$T_\chi = 4B\beta^2\sin^2\left(\gamma - \chi\frac{2\pi}{3}\right).$$

In the case $\gamma \equiv 0,\ \pi$ this expression is greatly simplified:

$$H = \frac{1}{2}B\dot{\beta}^2 + \frac{M^2}{6B\beta^2} + \frac{1}{2}C\beta^2.$$

Hence, according to Pauli' s prescription, the Schrödinger equation is written as

$$-\frac{h^2}{2B}\left\{\frac{1}{\beta^2}\frac{\partial}{\partial\beta}\beta^2\frac{\partial}{\partial\beta} + \frac{1}{3\beta^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]\right\}\Psi + \frac{1}{2}C\beta^2\Psi = E\Psi. \quad (2)$$

The action of an even number of extra-shell neutrons and protons is manifested in the fact that the expression for the potential energy changes: the "excess" nucleons deform the core, and the equilibrium position will be at $\beta = \beta_0$. A doubly magic nucleus corresponds to $\beta_0 = 0$.

Thus, for a deformed nucleus, instead of (2) we obtain

$$-\frac{h^2}{2B}\left\{\frac{1}{\beta^2}\frac{\partial}{\partial\beta}\beta^2\frac{\partial}{\partial\beta} + \frac{1}{3\beta^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]\right\}\Psi+$$

$$+\frac{1}{2}C(\beta-\beta_0)^2\Psi = E\Psi. \quad (3)$$

Equation (3) was given in paper ([5]). We shall seek the function Ψ in the form:

$$\Psi(\beta,\theta,\varphi) = \frac{u(\beta)}{\beta}Y_l^m(\theta,\varphi).$$

The wave function must not depend on the direction of the symmetry axis of the system; therefore l takes only even values. For $u(\beta)$ we have

$$-\frac{h^2}{2B}\left[\frac{d^2u}{d\beta^2} - \frac{l(l+1)}{3\beta^2}u\right] + \frac{1}{2}C(\beta-\beta_0)^2u = Eu. \quad (4)$$

For $l = 0$ this equation is solved exactly. If $l \neq 0$, the spectrum can be found approximately by expanding the expression

$$\frac{h^2}{2B}\frac{l(l+1)}{3\beta^2} + \frac{1}{2}C(\beta-\beta_0)^2 \quad (5)$$

in powers of β (the expansion is made about the point where (5) has a minimum) and restricting oneself to the second power of β. Then (4) is reduced to the equation for Hermite functions.

Fig. 1

The energy spectrum obtained from equation (4) is shown in Fig. 1. Along the abscissa is plotted the value $\beta_0 \sqrt[4]{BC/h^2}$, and along the ordinate—the energy in units $h\sqrt{C/B}$.

From the generalized model it is not yet possible to obtain correct absolute values for the spacings between the energy levels of nuclei, since the hydrodynamic considerations used to calculate the parameters B and M are far from reality. Therefore only the ratios of the spacings between levels and the spins of the levels can be compared with experiment. In order to determine the position of a nucleus on the diagram, the ratio E_2/E_1 was calculated, where E_1 and E_2 are the energies of the first and second excited levels, measured from the energy of the ground state. Then on the graph such a

the abscissa, to which the given ratio E_2/E_1 corresponds, and the sequence of spins of these levels observed experimentally. A question mark is placed in those cases where the spin of the level has not been determined experimentally.

We have considered all heavy and semi-heavy nuclei ($A > 70$) with a known spectrum of the first excited levels. In a large group of nuclei, the first two excited levels are approximately equidistant and, for positive parity, have spin 2. The level 0+, obtained from the theory and lying next to the lower 2+ level, is not observed in them. Such a discrepancy with experiment may possibly be explained by insufficient experimental accuracy, since the 0—0 transition, effected by means of conversion electrons, has low intensity compared with the 2—0 transition of nearby energy.

Let us note that nuclei with a small degree of deformation (Pb^{206}, Sr^{88}, Ge^{140}, and Po^{210}) do not fit into the proposed scheme. The experimental data are taken from works ([6–13]).

In conclusion, the author thanks Acad. I. E. Tamm and A. S. Davydov for valuable comments, and V. M. Strutinsky for his interest in the work.

Received
2 III 1957

CITED LITERATURE

[1] O. Bohr, *Problems of Contemporary Physics*, **9**, 1 (1955).
[2] O. Bohr, B. Mottelson, *Problems of Contemporary Physics*, **1**, 2 (1956).
[3] K. Ford, *Problems of Contemporary Physics*, **1**, 3 (1956).
[4] O. Bohr, *Problems of Contemporary Physics*, **1**, 1 (1956).
[5] L. Wilets, M. Jean, Phys. Rev., **102**, 788 (1956).
[6] R. P. Scharenberg et al., Phys. Rev., **101**, 689 (1956).
[7] J. J. Kraushaar et al., Phys. Rev., **101**, 139 (1956).
[8] D. E. Alburger, B. J. Toppel, Phys. Rev., **100**, 1357 (1955).
[9] R. E. Rowland, Phys. Rev., **99**, 757 (1955).

[10] N. Benezer et al., Phys. Rev., **101**, 1030 (1956).
[11] J. D. Kurbatov et al., Phys. Rev., **98**, 674 (1955).
[12] H. Motz, Phys. Rev., **104**, 1353 (1954).
[13] L. J. Baggerly et al., Phys. Rev., **100**, 1364 (1955).

Note: Figure translations are in progress. See original paper for figures.