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Abstract

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Hydromechanics

Yu. L. Yakimov

An Asymptotic Solution with Three Arbitrary Functions of the Equations of One-Dimensional Unsteady Gas Motion

(Presented by Academician L. I. Sedov, 29 IV 1957)

A solution of the system of equations describing the unsteady motion of an ideal gas with spherical ($\nu = 3$) symmetry,

$$\begin{gathered}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0,\\ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial r} + (\nu - 1)\frac{\rho u}{r} = 0,\\ \frac{\partial}{\partial t}\left(\frac{p}{\rho^{\gamma}}\right) + u\frac{\partial}{\partial r}\left(\frac{p}{\rho^{\gamma}}\right) = 0,\end{gathered} \tag{1}$$

where u is the velocity; ρ the density; p the pressure; t the time; r the radius; γ the adiabatic exponent, is

$$\begin{gathered}p = p_0\left\{1 + \frac{1}{r}[F + f] + \frac{1}{r^2}\left[\left(\frac{1}{8} - \frac{7}{8\gamma}\right)F^2 + \right.\right.\\ \left.\left. + \left(-\frac{1}{8\gamma\sqrt{\gamma}} - \frac{3}{4\gamma} - \frac{1}{8\gamma} - \frac{1}{4}\right)f^2 + \frac{2}{\gamma}Ff\right] + \sum_{k=3}^{\infty}\sum_{i=0}^{k-1} p_{ki}\frac{\ln^i r}{r^k}\right\};\\ \rho = \rho_0 S\left\{1 + \frac{1}{r}\left[\frac{1}{\gamma}F + \frac{1}{\gamma}f\right] + \frac{1}{r^2}\left[\left(-\frac{3}{8\gamma} - \frac{3}{8\gamma^2}\right)F^2 + \left(\frac{3}{\gamma^2} - \frac{1}{\gamma}\right)Ff + \right.\right.\\ \left.\left. + \left(-\frac{1}{8\gamma^2\sqrt{\gamma}} - \frac{1}{4\gamma^2} - \frac{1}{8\gamma\sqrt{\gamma}} - \frac{1}{4\gamma}\right)f^2\right] + \sum_{k=3}^{\infty}\sum_{i=0}^{k-1} \rho_{ki}\frac{\ln^i r}{r^k}\right\};\end{gathered} \tag{2}$$

$$u = \sqrt{\frac{p_0}{\rho_0 S}}\left\{\frac{1}{r}\left[\frac{1}{\sqrt{\gamma}}F + \frac{1}{\sqrt{\gamma}}f\right] + \frac{\ln r}{r^2}\left[\frac{1}{4\sqrt{\gamma}}\left(\frac{1}{\gamma} + 1\right)(-F^2 + f^2)\right]\right.$$

$$+\frac{1}{r^2}\left[\left(-\frac{1}{8\sqrt{\gamma}}-\frac{9}{8\gamma\sqrt{\gamma}}\right)F^2-\frac{1}{\sqrt{\gamma}}\int_{\xi_{10}}^{\xi_1}F\,d\xi_1+\left(\frac{1}{8\gamma^2}+\frac{1}{8\sqrt{\gamma}}+\frac{1}{8\gamma}\right)f^2+\right.$$

$$+\frac{1}{\sqrt{\gamma}}\int_{\xi_{20}}^{\xi_2}f\,d\xi_2+\left(\frac{1}{2}-\frac{1}{2\gamma}+\frac{2}{\gamma\sqrt{\gamma}}\right)Ff-\frac{8}{\sqrt{\gamma}(\gamma+1)}\int_{\frac{1}{2}(\xi_{10}+\xi_{20})}^{\frac{1}{2}(\xi_1+\xi_2)}Ff\,d(\xi_1+\xi_2)\Bigg|_{(\xi_1-\xi_2)=\text{const}}$$

$$\left.-\frac{1}{4\sqrt{\gamma}}(F-f)\frac{d\ln S}{d\xi_3}\right]+\sum_{k=3}^{\infty}\sum_{i=0}^{k-1}u_{ki}\frac{\ln^i r}{r^k}\Bigg\},$$

where p_0 and ρ_0 are arbitrary constants; $F(\xi_1)$, $f(\xi_2)$, $S(\xi_3)$ are arbitrary functions of their arguments; ξ_1, ξ_2, ξ_3 are characteristic variables.

of system (1), related to r, t by the formulas

$$\mu_1 d\xi_1 = dr-(a+u)\,dt, \qquad \mu_1=\left(\frac{1}{2}+\frac{1}{2\gamma}\right)F'_{\xi_1}\ln r+1+\sum_{k=1}^{\infty}\sum_{i=0}^{k}\mu_{1ki}\frac{\ln^i r}{r^k};$$

$$\mu_2 d\xi_2 = dr-(-a+u)\,dt, \qquad \mu_2=\left(\frac{1}{2}+\frac{1}{2\gamma}\right)f'_{\xi_2}\ln r+1+\sum_{k=1}^{\infty}\sum_{i=0}^{k}\mu_{2ki}\frac{\ln^i r}{r^k};$$

$$\mu_3 d\xi_3 = dr-u\,dt, \qquad \frac{1}{\mu_3}=\frac{1}{r^2}+\sum_{k=3}^{\infty}\sum_{i=0}^{k-1}\mu_{3ki}\frac{\ln^i r}{r^k}, \tag{3}$$

where the quantities $p_{ki}, \rho_{ki}, u_{ki}, \mu_{1ki}, \mu_{2ki}$, and μ_{3ki} are functionals of $F(\xi_1)$, $f(\xi_2)$, $dS(f_3)/d\xi_3$, which vanish if the functions $F(\xi_1)$ and $f(\xi_2)$ are simultaneously zero. These functionals are determined successively for arbitrary values of k and i by means of a finite number of operations.

Let us make several remarks concerning the method of obtaining this solution and the method of finding the subsequent terms of the solution.

If, from the equalities (3), we eliminate dt, and in system (1) make the change of variables according to the formulas

$$\begin{aligned}\frac{\partial}{\partial r}&=\frac{1}{\mu_1}\frac{\partial}{\partial\xi_1}+\frac{1}{\mu_2}\frac{\partial}{\partial\xi_2}+\frac{1}{\mu_3}\frac{\partial}{\partial\xi_3}+\frac{\partial}{\partial r};\\ \frac{\partial}{\partial t}&=-\frac{a+u}{\mu_1}\frac{\partial}{\partial\xi_1}-\frac{-a+u}{\mu_2}\frac{\partial}{\partial\xi_2}-\frac{u}{\mu_3}\frac{\partial}{\partial\xi_3},\end{aligned} \tag{4}$$

then, instead of system (1), we obtain a system consisting of: a) three equations containing six functions $p, \rho, u, \mu_1, \mu_2, \mu_3$, and four variables ξ_1, ξ_2, ξ_3, r, which have been obtained from equations (1) by replacing the derivatives in them according to the formulas (4); b) three equations for the integrating factors, expressing the condition that dt is an exact differential, with the six functions $\mu_1, \mu_2, \mu_3, p, \rho, u$ and four variables ξ_1, ξ_2, ξ_3, r; c) the three equalities (3), which serve to determine ξ_1, ξ_2, ξ_3 as functions of r and t.

Any solution of the first six equations, containing six unknown functions and four variables ξ_1, ξ_2, ξ_3, r, which we here regard as formally independent, will satisfy the original system (1) if, instead of ξ_1, ξ_2, ξ_3, one substitutes their expressions in terms of r and t, found from the equalities (3).

From formulas (2) it is seen that the solution is sought in the form of a series; moreover, collecting in the equations the terms with identical powers of $\ln r$ and r, and equating them to zero, we obtain equations for determining the unknown functions.

The functions satisfying these equations and possessing the listed properties are found successively for any number k, i by means of a finite number of operations. In (2) the first three terms are given.

In order for the solution of the problem to be complete, it is necessary to find three finite relations among the variables $\xi_1, \xi_2, \xi_3, r, t$. Two of the variables ξ_1, ξ_2, and ξ_3 drop out if two of the functions $F(\xi_1)$, $f(\xi_2)$, and $dS(\xi_3)/d\xi_3$ vanish. In this case formulas (2) give a solution with two independent variables.

The construction of the asymptotic solution considered is analogous to the construction of the solution with two arbitrary functions $F(\xi_1)$ and $f(\xi_2)$, examined in more detail in (^{2}), where it is used to refine the asymptotic laws of decay of spherical shock waves.

An analogous result can be obtained for other equations of state and for the case of cylindrical symmetry ($\nu = 2$).

Moscow State University
named after M. V. Lomonosov

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REFERENCES

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Note: Figure translations are in progress. See original paper for figures.