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Abstract

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FLOW PAST ELLIPSES BY A GAS STREAM AT THE SPEED OF SOUND

(Presented by Academician A. A. Dorodnitsyn, 23 X 1956)

In using electronic computing machines to solve certain nonlinear problems of gas dynamics, one numerical method proposed by A. A. Dorodnitsyn ([1]) has proved effective. By this method, which approximately reduces two-dimensional problems to ordinary differential equations, several examples have been calculated, in particular, the flow past an airfoil and a body of revolution at subsonic speed ([2]). Here, in a similar way, a calculation is carried out for the flow of a sonic gas stream past ellipses.

Let a stream at the speed of sound approach along the major axis of an ellipse. As is known, in this case there are two semi-infinite characteristics beginning on the body and dividing the entire flow into two regions. We shall calculate the mixed potential motion in region I (upstream of the characteristics). It can be constructed independently of the motion in region II and is described by the equations of continuity and irrotationality. If dimensionless quantities are introduced by referring lengths to one half of the focal distance, velocities to the maximum velocity in the gas, and density to the stagnation density, then these equations in elliptic coordinates ξ, η will be

$$\frac{\partial \chi}{\partial \xi} + \frac{\partial \omega}{\partial \eta} = 0, \qquad \frac{\partial \lambda}{\partial \xi} - \frac{\partial \mu}{\partial \eta} = 0; \tag{1}$$

here $\chi = H\rho u;\ \omega = H\rho v;\ \lambda = Hv;\ \mu = Hu;\ H = \sqrt{\operatorname{sh}^2 \xi + \sin^2 \eta};\ u, v$ are the velocity components along hyperbolas and ellipses; $\rho = (1 - u^2 - v^2)^{\frac{1}{\kappa - 1}}$ is the density; κ is the adiabatic exponent (for air $\kappa = 1.4$).

By symmetry it is sufficient to consider only the half-plane $0 \le \eta \le \pi$. For the boundary characteristic of the first family we have:

$$\frac{d\eta_1}{d\xi} = \frac{u_1 + v_1 \operatorname{ctg} \alpha_1}{-v_1 + u_1 \operatorname{ctg} \alpha_1}, \tag{2}$$

where

$$\alpha_1 = \arcsin\sqrt{\frac{1-u_1^2-v_1^2}{u_1^2+v_1^2}\,\frac{\kappa-1}{2}},$$

$$\frac{du_1}{d\xi} = \frac{1}{v_1+u_1\operatorname{ctg}\alpha_1}\left[(u_1-v_1\operatorname{ctg}\alpha_1)\frac{dv_1}{d\xi}+\frac{u_1^2+v_1^2}{2H_1^2}\left(\sin 2\eta_1+\frac{d\eta_1}{d\xi}\operatorname{sh}2\xi\right)\right]. \tag{3}$$

We shall agree to denote quantities on the axis $\eta = 0$ by the subscript 0, quantities on the characteristic by the subscript 1, and quantities on an intermediate line by the subscript n,

$$\eta_n = \frac{N-n+1}{N}\eta_1 \qquad (N \text{ is the number of the approximation, } n = 1, 2, \ldots, N).$$

The boundary conditions of the problem are as follows. On the axis $v_0 = 0$; on the ellipse being flowed past, $\xi_{\text{ob}} = \operatorname{ar\,th}\delta$ (δ is the relative thickness), $u_n = 0$; at infinity, for a sonic stream,

$$\eta_1 = \frac{\pi}{2}, \qquad u_n = \sqrt{\frac{\kappa-1}{\kappa+1}}\cos\eta_n, \qquad v_n = -\sqrt{\frac{\kappa-1}{\kappa+1}}\sin\eta_n. \tag{4}$$

Integrating (1) with respect to η from 0 to η_n, we form the integral relations

$$\frac{d}{d\xi}\int_0^{\eta_n}\chi\,d\eta - \chi_n\frac{d\eta_n}{d\xi} + \omega_n = 0,$$

$$\frac{d}{d\xi}\int_0^{\eta_n}\lambda\,d\eta - \lambda_n\frac{d\eta_n}{d\xi} - \psi_n + \psi_0 = 0. \tag{5}$$

The solution of the problem can be carried out with various degrees of approximation. In the N-th approximation, introducing $N-1$ intermediate lines η_n, we form $2N$ integral relations (5) for $n = 1, 2, \ldots, N$. In doing so, the integrand functions in (5) will be represented approximately by interpolation polynomials of the form

$$\chi = \sum_{k=0}^{N} a_k \cos k\frac{\pi}{2}\frac{\eta}{\eta_1}, \qquad \lambda = \sum_{k=0}^{N} b_k \sin k\frac{\pi}{2}\frac{\eta}{\eta_1}, \tag{6}$$

where a_k, b_k are linear functions of χ_n, λ_n. With these approximations all boundary conditions can be satisfied exactly.

Substituting (6) into (5), in the N-th approximation, together with (2) and (3), we obtain a system of $2N+2$ ordinary differential equations for the unknown values η_1, u_n, v_n. The equations following from (5) are written in the form:

$$\frac{d\eta_1\chi_n}{d\xi} = c_n\frac{d\eta_1\chi_1}{d\xi} + \sum_{i=1}^{N} d_{ni}\left(\chi_i\frac{d\eta_i}{d\xi} - \omega_i\right) \quad (n = 0, 2, 3, \dots, N),$$

$$\frac{d\eta_1\lambda_n}{d\xi} = \sum_{i=1}^{N} e_{ni}\left(\lambda_i\frac{d\eta_i}{d\xi} + \mu_i - \mu_0\right) \quad (n = 1, 2, \dots, N), \tag{7}$$

where c_n, d_{ni}, e_{ni} are numerical coefficients which, for example, for $N = 3$ are given in Table 1.

Table 1

n	d_{ni}, $i=0$	d_{ni}, $i=2$	d_{ni}, $i=3$	e_{ni}, $i=1$	e_{ni}, $i=2$	e_{ni}, $i=3$
1	0.6576	2.0962	−0.8542	6.5262	0.5750	−0.0889
2	−2.0459	−0.0032	3.0503	−14.8662	3.7165	0.9958
3	5.3700	−2.6162	−0.8542	18.4938	−8.0080	2.8096
c_n	−0.0837	−0.2221	0.1054			

The resulting system is integrated numerically, starting from the body being flowed around, the ellipse ξ_{ob}, and it is convenient to pass to the new independent variable $t = e^{-\xi}$. Here a boundary-value problem arises, since in the N-th approximation at ξ_{ob} all $u_n = 0$, while the N values v_n are chosen from conditions (4) at infinity ($t = 0$). The initial value η_1 is determined from v_1 by (3), which in the plane case reduces to a transcendental equation. After integrating the system with the approximations (6), one can construct the velocity field and, in particular, the sonic line $\eta_*(\xi)$.

Using the BESM electronic computer, by this method there were calculated an ellipse $\delta = 0.2$ and a circle, the selection of the values v_n being carried out partly with the aid of interpolation by the machine itself. Calculations for the circle were made for $N = 1, 2, 3$. The results of the calculations are shown in Fig. 1, where the distribution of the Mach number M on the body being flowed around —the circle of radiu-

for $r = 1$, and in Fig. 2, where the dependence of the Mach number on $t = 1/r$ is plotted for the axis (M_0) and for the characteristic (M_1). The third approximation, as may be judged from these graphs, gives sufficient accuracy.

Fig. 1
Fig. 1

Fig. 2
Fig. 2

Table 2 gives the distribution of the number M on the ellipse being flowed around and on the circle.

Table 2

δ	η_1	η/η_1	0	0.05	0.1	0.2	0.4	0.6	0.8	1.0
0.2	1.263	M	0	0.260	0.470	0.715	0.889	0.981	1.083	1.203
1.0	1.25[[unclear: final digit]]	M	0	0.067	0.43[[unclear: final digit]]	0.[[unclear]]	0.5[[unclear]]	0.9[[]]	[[unclear: value]]	1.[[unclear]]

CORRECTIONS

1. On p. 510, line 27, it should read: $i = 1, 2, \dots, N$.
2. On p. 518, in formula (5), the first equation should be read as:

$$\frac{d}{d\xi}\int_0^{\eta_n} \chi \, d\eta - \chi_n \frac{d\eta_n}{d\xi} + \omega_n = 0.$$

3. In the first column of Table 1 the values of the index i are given.
4. On p. 520, the third paragraph should read:

The method of A. A. Nikol' skii ([1]) makes it possible to formulate the problem under consideration for functions on the characteristic BC.

5. On p. 522, the equation in line 7 should be read as:

$$\frac{dr}{d\psi} = -\frac{\omega(a)\varphi(\psi)}{\sqrt{\chi r}} \sin(\vartheta - a),$$

$$Vr^z + F_\theta^z \qquad I_\eta V r^z + F_\theta^z$$

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CITED LITERATURE

1. A. A. Dorodnitsyn, *Proceedings of the 3rd All-Union Mathematical Congress*, **2**, 1956.

2. O. N. Katskova, P. I. Chushkin, Yu. D. Shmyglevskii, Report at the conference "Paths of development of Soviet mathematical machine-building and instrument-making," 1956.

Note: Figure translations are in progress. See original paper for figures.