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Abstract

Full Text

THEORY OF ELASTICITY

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A THEORY OF PLASTICITY TAKING INTO ACCOUNT THE BAUSCHINGER EFFECT

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1. A theory of plasticity of quasi-isotropic bodies is proposed, based on the following relations between plastic strains and stresses:

$$d\varepsilon_{ij}^p = \bar{\sigma}'_{ij}\, df(\bar{T}), \qquad s_{ij} = 2g(\Gamma)\varepsilon_{ij}^p; \tag{1}$$

$$\bar{\sigma}_{ij} = \sigma_{ij} - s_{ij}, \qquad \bar{\sigma}'_{ij} = \bar{\sigma}_{ij} - \frac{1}{3}\sigma\delta_{ij}, \qquad \sigma = \sigma_{ii}; \tag{2}$$

$$\bar{T} = \sqrt{\frac{1}{2}\bar{\sigma}'_{ij}\bar{\sigma}'_{ij}}, \qquad \Gamma = \sqrt{\frac{1}{2}\varepsilon_{ij}^p\varepsilon_{ij}^p}. \tag{3}$$

We shall call s_{ij} the **tensor of residual stresses**, and $\bar{\sigma}_{ij}$ the **tensor of active stresses**. The meaning of the latter term becomes clear from the fact that in the proposed theory the condition for activity of deformation is expressed by the inequality

$$d\bar{T} > 0, \tag{4}$$

and the equation of the yield boundary has the form

$$\bar{T} = \text{const}, \tag{5}$$

i.e., in the new theory the active stresses play the same role that the actual stresses σ_{ij} play in the flow theory. The equation

$$T = \sqrt{\frac{1}{2}\sigma'_{ij}\sigma'_{ij}} \tag{6}$$

in the five-dimensional space introduced in work ([1]) is a sphere with center at the origin. Equation (5), however, in the same space, is a sphere with center at the point determined by the tensor S_{ij}. Thus, in the theory under consideration the yield boundary has the same form as in the flow theory; however, its center, in the process of deformation, is displaced according to the law given by formulas (1). In this way the Bauschinger effect is taken into account.

As for the stresses S_{ij}, then, besides the fact that they determine the position of the center of the region of elastic deformations, the following may be said about them:

A. They are equal to zero at the moment of appearance of the first plastic deformations (since at that moment the region of elastic deformations is symmetric with respect to the origin).

B. They depend, according to (1), on the plastic deformations by the principle of elastic interaction.

C. By virtue of this, they remain in the body after unloading (which explains the name adopted above for them).

2. Let us consider the following mechanical model, which explains the proposed relations, restricting ourselves, for simplicity and clarity, to the consideration of the special case in which only σ_{xx}, σ_{xy} are different from zero.

If we introduce the notation

$$X = \frac{2}{3}\sigma_{xx}, \qquad Y = \frac{2}{\sqrt{3}}\sigma_{xy}$$

and consider, in the X, Y plane, a body (see Fig. 1) subjected to: a) external forces X, Y, applied by means of springs A and B; b) forces X_1, Y_1, arising during displacements of the body owing to deformation of springs C and D; c) a friction force F, generally speaking variable and continuously increasing in the process of displacement of the body, then the equilibrium condition of this body will be written in the form

$$\sqrt{(X - X_1)^2 + (Y - Y_1)^2} = F,$$

and the direction of its displacement in the plane when equilibrium is disturbed will always be directed along the normal to the circle $F = \text{const}$ with center at X_1, Y_1.

Fig. 1

The mechanical model described corresponds to formulas (1) (in the two-dimensional case), where the function g in them characterizes the compliance of springs C and D; the stresses s_{ij} are the forces in these springs; ε_{ij}^p are the components of the displacement of the body; σ_{ij} are the forces acting on springs A and B; $\bar{\sigma}_{ij}$ are the differences between these forces and the forces in springs C, D. It is precisely these differences (taken together) that balance the friction force when the body is in a position of neutral equilibrium. As for the function $f(\bar{T})$, it characterizes the law of variation of dry friction. If all hardening is attributed to an increase of dry friction, then $g = 0$, $s_{ij} = 0$, and we arrive at the theory of flow in its usual formulation. If, however, it is assumed that the Bauschinger effect takes place, i.e., that part of the hardening is reversible, then we arrive at theory (1).

3. Above we neglected the influence of the mean normal stress on the yield criterion. When this influence is taken into account, the analogy between dry friction and resistance to plastic deformation leads to certain contradictions ([2]). This difficulty can be avoided in a way entirely analogous to how it is avoided in the theory of flow when the Mises plasticity boundary is replaced by some other boundary.

4. A simpler variant of the theory set forth, analogous to Reuss' s theory, deserves attention; its formulas have the form

$$d\varepsilon_{ij}^{p} = \bar{\sigma}'_{ij}\, d\lambda, \qquad s_{ij} = 2g_0 \varepsilon_{ij}^{p}, \tag{7}$$

$$T = \frac{1}{\sqrt{3}} \sigma_T, \tag{8}$$

where σ_T is the yield limit in simple tension; g_0 is a constant characterizing hardening.

According to (8), the yield boundary moves as a rigid body—without changing its dimensions. Formulas (7) and (8) correspond to the case in which all hardening is considered elastic, and its law is assumed linear. Materials in which the hardening effect is completely reversible may be called materials with an ideal Bauschinger effect. This variant was proposed earlier by A. Yu. Ishlinsky ([7]).

5. Comparison of the results of the proposed theory with experimental data shows that it makes it possible to interpret a number of experimental facts,

not explained either by flow theory or by the theory of small plastic deformations, among which, in particular, are: a) the Bauschinger effect; b) the effect of noncoincidence of the principal axes of the stress tensor and the tensor of increments of plastic deformations, noted in ([3]); c) the fact that experimental curves, as a rule, pass between the curves obtained from flow theory and from the theory of small plastic deformations; d) the fact that, when going around the yield boundary accepted in flow theory, residual deformations are observed experimentally ([4]); e) the fact that a substantial part of the work expended on plastic deformation (of the order of 10%) is not converted into heat ([5]).

It should be noted that even the simplest version of the theory, assuming that the Bauschinger effect is ideal and the hardening is linear (7), (8), gives a picture of plastic deformations that agrees qualitatively (and sometimes quantitatively) with experimental results.

6. Elements of the proposed theory (in implicit form) had already been given in ([6]). However, the absence in that work of any physical analysis of the results obtained prevented the author from introducing the concepts of active and residual stresses, which, ultimately, was also reflected in the final formulas; in structure they are more complicated than (1) or (7), (8), and, in our opinion, are less consistent.

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CITED LITERATURE

1. A. A. Ilyushin, *Prikladn. matem. i mekh.*, **18**, issue 6 (1954).
2. D. C. Drucker, *Quart. Appl. Math.*, **7**, No. 4 (1950).
3. M. Feigin, Collection of Translations on Mechanics, issue 3, 1956.
4. L. Hu, J. Marin, *Trans. Am. Soc. Mech. Eng.*, **78**, No. 3 (1956).
5. G. S. Shapiro, *Prikladn. matem. i mekh.*, **17**, issue 1 (1953).
6. D. C. Drucker, *J. Franklin Inst.*, **251**, June (1951).
7. A. Yu. Ishlinskii, *Ukr. matem. zhurn.*, **6**, No. 3 (1954).

Note: Figure translations are in progress. See original paper for figures.