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Abstract

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OPTICAL SYSTEMS WITH PHASE LAYERS

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At the present time two groups of optical systems that form images of objects are known. The feature distinguishing these groups from one another is the course of the variation of the optical path length with the aperture angle. In ordinary, well-known optical systems consisting of lenses and mirrors, the optical path length from a point of the object to its image is constant or nearly constant along all rays intersecting any point of the entrance pupil. For the second group of less well-known and not widely used optical systems, an example of which is the Soret zone plate or its modifications (Wood' s zone plate [1], etc.), the path length is not constant and may change by any number of wavelengths; but measures are taken so that the optical path length for all points of the working part of the pupil can be represented in the form $a + k\lambda$, where a is a quantity constituting, insofar as possible, a small part of the wavelength; k is any integer; λ is the wavelength.

It may be noted that "ordinary" optical systems are a special case ($k = 0$) of the second group. However, in certain respects the properties of the first group differ so much from those of the second that merging these groups into one would be inadvisable from a practical point of view.

Systems of the second type have not found practical application for three reasons: 1) all systems known at present collect at the point no more than 40% of the incident energy; 2) they possess a very considerable and, in principle, uncorrectable chromatic aberration; 3) because of the enormous number of zones (of the order of a thousand) required to obtain sufficient optical power, great technological difficulties arise in making such plates.

The only rational attempt to use a zone plate made by photographing Newton' s interference rings belongs to O. Myers [2], who used it as the objective of a spectrograph; in this case the presence of large chromatic aberration ceases to be an obstacle. However, the image quality proved very poor, and the exposure time required was 16 times greater than for a simple lens with the same relative aperture.

Experiments carried out by S. M. Rayskii [3] with the aid of dark rings deposited on spherical reflecting surfaces experimentally proved the existence of diffraction foci.

Fig. 1

Figure 1: Fig. 1

The present article aims to draw attention to a number of possibilities inherent in systems consisting of lenses and modified zone plates. As will be shown below, all the above-mentioned shortcomings of zone plates can either be completely eliminated or used to compensate the residual chromatic aberration (secondary spectrum) of ordinary optical systems.

As a result, it is possible to obtain very good correction of the secondary spectrum of astronomical and other long-focus systems, for which

this aberration reaches very large values. At the same time, it turns out that the number of interfering beams is comparatively small, of the order of 30–40 for a focal length of 1000–2000 mm and a relative aperture of 1 : 10.

The first and principal disadvantage of the zone plates of Soret or Wood is the scattering of light energy among images of different orders; this disadvantage follows from the fact that the optical path length connecting a point object and its image is not constant. However, it can be made constant by giving the zone plate a special form, determined by the condition that over the whole zone the optical path length is constant, while on passing from one zone to the next there is a phase jump of 2π. Among the innumerable possible forms of plate profiles one may indicate several, shown schematically in Fig. 1. The maximum thickness d_0 is equal to $\lambda_0/(n_0-1)$, where n_0 and λ_0 are, respectively, the refractive index and the wavelength for which the plate is calculated; in what follows we shall call the latter a phase plate. The thickness of the phase layers varies according to the law $d = d_0 - a(h^2 - h_k^2)$, where a is a certain constant depending only on the position of the object and image and on the wavelength; h is the height of intersection of the ray; h_k is the height of the edge of the k-th zone. Here $h_k < h < h_{k+1}$.

Fig. 1

The phase plates described can be realized by applying to one of the lens surfaces thin layers whose thickness varies according to the indicated law, or by producing depressions by a special polishing method (V. A. Savin).

The focal length f_0' of a phase plate, i.e. the quantity determined by the equation $\frac{1}{s'} - \frac{1}{s} = \frac{1}{f_0'}$, where s and s' are the distances from the plate to the object and to its image for the principal wavelength λ_0, is calculated by the formula

$$f_0' = \frac{h_1^2}{2\lambda_0}. \tag{1}$$

For wavelengths different from λ_0, no such abscissa s' can be found at which 100% of the light energy incident from the object would be collected. The abscissa of the point S', where the maximum of the light energy is collected, is determined from the equation

$$\frac{1}{s'_\lambda} - \frac{1}{s'_0} = \frac{1}{f'_\lambda}, \quad \text{where } f'_\lambda = \frac{h_1^2}{2\lambda}, \tag{2}$$

known from the theory of zone plates.

From this equation follows the enormous value of the chromatic aberration $\Delta s'$ of zone, and also phase, plates, namely:

$$\Delta s' = -2{s'}^2 \frac{\Delta\lambda}{h_1^2} = -\frac{s_1^2}{f'_0}\frac{\Delta\lambda}{\lambda}. \tag{3}$$

The chromatic aberration in the usually considered spectral region C—F (656-486 mμ) is equal to 1/3 of the value ${s'}^2/f'$, whereas for lenses it is equal to 1/60—1/64 for ordinary crowns, i.e., 20 times smaller.

Calculations show that at the focus corresponding to the wavelength $\lambda \neq \lambda_0$, the energy can be determined by the formula:

$$E(\lambda) = \left(\frac{\lambda_0}{\lambda}\right)^2 \frac{\sin^2 B/2}{(B/2)^2}, \qquad \text{where } B = 2\pi\left[1 - \frac{\lambda_0}{\lambda}\frac{n-1}{n_0-1}\right], \tag{4}$$

if the energy at the focus of the rays λ_0 is taken as unity. If, for example, $\lambda_0 = 510$ mμ is taken, then for $\lambda = 380$ mμ $E = 0.886$, for $\lambda = 600$ mμ $E = 0.671$, and for $\lambda = 700$ mμ $E = 0.413$. Thus, the phase plate represents a certain analogy with a light filter; it absorbs the extreme regions of the spectrum more than the middle ones; however, if one takes into account the spectral sensitivity of the receiver (the eye, photographic plate, etc.), the losses caused by this phenomenon are quite small: for the eye they do not exceed 2-3%.

The question naturally arises as to whether the indicated phenomenon causes scattering of light and a deterioration of the resolving power of the system. To clarify this, it is necessary to compute the energy distribution in the plane containing the image of a point and located according to formula (2).

Rather complicated calculations, based on applying the Huygens–Fresnel principle with Kirchhoff' s supplement and using the properties of Bessel functions, lead to the following formula:

$$E = \frac{4J_1^2(n)}{n^2}\frac{\sin^2 B/2}{(B/2)^2} + \frac{1}{q^2}\left\{\frac{J_1(n)}{6}\left[\frac{n}{2} - J_1(n)\right]\left[\frac{12\sin B}{B^3} + \left(1 - \frac{12}{B^2}\right)\frac{\sin^2 B/2}{(B/2)^2}\right]\right.$$

$$+ [J_0(n) - 1]^2 \left[\left[\cos B/2 - \frac{\sin B/2}{B/2} \right] \frac{1}{B^2} + \cdots \right] \Bigg\}, \tag{5}$$

where q is the number of interfering zones of the phase plate; $n = \dfrac{2\pi}{\lambda} z \sin\omega'$; z is the distance from the axis to the point for which the energy is determined; ω' is the aperture angle of the beam. Since the value of q in cases of interest is not less than 30–40, and the expressions in double brackets have small numerical values for all values of B, one may regard, with quite sufficient accuracy,

$$E = \frac{4J_1^2(n)}{n^2} \frac{\sin^2 B/2}{(B/2)^2}. \tag{6}$$

Thus, the distribution is the same as for an ideal optical system with the same aperture angle, but in our case there appears the factor $\left(\dfrac{\sin B/2}{B/2}\right)^2$, which depends on the wavelength of the light under consideration. This formula confirms the "filter-like" action of the phase plate, although it should be noted that, unlike ordinary light filters, the action of a phase plate changes depending on a number of quantities (for example, the position of the installation plane).

The image quality of points lying outside the optical axis of the system can be determined on the basis of the third-order aberrations of systems of lenses with a phase plate. These aberrations are readily obtained from calculating the dependence of the length of the optical path on the coordinates on the phase plate. From the expressions for the aberrations, not given here (because of lack of space), there follow several curious properties of phase plates:

1) The spherical aberration is determined by the coefficient c of the expansion into a series of the thickness of the phase plate as a function of h:

$$d(h) = \frac{\lambda_0}{n_0 - 1} - \frac{h^2}{2f_0'(n_0 - 1)} + \frac{h^4}{c^3} + \cdots.$$

2) The curvature coefficient of the image surface is equal to zero for any optical power of the phase plate.

3) The radius of the surface on which the phase layers are deposited has no effect on the third-order aberrations.

4) It is almost impossible to influence the remaining aberrations of the phase plate. However, for the case considered above of an apochromat consisting of two lenses with a phase plate, these aberrations are vanishingly small. In the latter case it is possible not only to correct the secondary spectrum, but also the sphero-chromatic aberration, which is impossible in ordinary two-lens objectives.

Applications. In addition to the possibility already described of obtaining a two-lens objective with a phase surface, possessing the properties of highly perfect correction of chromatic aberrations and especially suitable for astronomical purposes, one may use existing refractor objectives and, by adding to them a lens calculated in a definite way, with a phase surface, shortening its focal length by approximately 1/7 of its value, obtain an apochromat with high quality of the central image.

Other applications are also possible, but their development has not yet been completed.

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Note: Figure translations are in progress. See original paper for figures.