ON THE FORCE ACTION OF THE “SECOND” FORM OF HYDRODYNAMIC MOTION ON PLANE BODIES

1957

SovietRxiv

Abstract

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HYDROMECHANICS

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ON THE FORCE ACTION OF THE “SECOND” FORM OF HYDRODYNAMIC MOTION ON PLANE BODIES

(DYNAMICS OF PLANE SEPARATED FLOWS)

(Presented by Academician A. A. Dorodnitsyn, 23 IV 1957)

The paper (1) is devoted to the investigation of the flow of an ideal fluid around bodies when, near them, a “second” form of motion develops, associated with the formation in the flow of surfaces γ of tangential discontinuity of velocity. In the present work, as in (1), the complete velocity field is regarded as the sum of the principal field E_1, corresponding to the flow around the body without the formation of the surfaces γ, and an additional field E_2; by Γ we denote the total circulation of a piece of the surface γ cut off by the point of it under consideration; $w = \varphi + i\psi$, $w_1 = \varphi_1 + i\psi_1$, $w_2 = \varphi_2 + i\psi_2$ are, respectively, the complex potentials of the fields E, E_1, E_2; t is time; p is pressure; ρ is density. Here the formulas of (1) are used in its notation. In (1), two cases of motion were considered: case I and case II, each with the corresponding subcases. Here we shall refer to them and shall consider two further cases III and IV, with the corresponding subcases, which already pertain to the flow around bodies of finite dimensions.

Let us consider, in the plane $z = x + iy$, an arbitrary change of an impermeable closed contour s, under the condition $dw/dz = v_0(t)e^{-i\alpha(t)}$ for $z = \infty$. Suppose that at $t = 0$ there are no vortices in the fluid, while for $t > 0$ vortex surfaces γ are formed near the body. Denote by X, Y, respectively, the components along the axes x, y of the resultant pressure force on the body. We shall denote by $s + \gamma$ the liquid closed contour consisting of the contour of the body plus each of the two sides of each of the lines γ. Applying the known formulas for the forces acting on a liquid contour (2), and using the fact that the residue of dw_2/dz near the point $z = \infty$ is zero, we obtain, despite the nonlinearity of the problem, the simple superposition rule

$$\frac{1}{\rho}(-Y_2 + iX_2) = \frac{d}{dt}\oint_{\gamma+s} z\,dw_2, \tag{A}$$

where $X_2 = X - X_1$, $Y_2 = Y - Y_1$; X_1, Y_1 are the forces in the flow E_1; X_2, Y_2 are the additional forces associated with the formation of γ. On s one may

Fig. 1

Figure 1: Fig. 1

always put $\psi_2 = 0$. Let on s also $\psi_1 = 0$—this will take place in the flow about fixed bodies of unchanged form and plane plates of variable width. Suppose, moreover, that the body moves along the y-axis and is symmetric with respect to it. The residue of w_2 near the point $z = \infty$ is equal to the residue in the w_1-plane near $w_1 = \infty$ of the function $w_2\, dz/dw_1$, and this latter is equal to the residue near $w_1 = \infty$ of the function w_2, multiplied by $-iv_0(t)$. This makes it possible, introducing the quantity Γ, to rewrite the first of equalities (A) in the form

$$X_2 = 0; \qquad Y_2 = 2\rho \frac{d}{dt}\left\{ \frac{1}{v_0(t)} \int_{\gamma_l} \psi_1 \, d\Gamma \right\}. \tag{1}$$

Here γ_l are the lines γ lying for $x < 0$.

III. w_1 coincides with the potential of flow past, along the y-axis, a body of characteristic dimension l, of unchanged shape, with angular points, symmetric with respect to the y-axis. Let $dw_1/dz = -iv_0(t)$ at $z = \infty$. The flow is considered for small

$$\varepsilon = \frac{1}{l}\int_0^t v_0(t)\, dt$$

(Fig. 1). Near each angle that is convex into the stream and is flowed around by the stream, for $t > 0$ the second form of motion will begin to develop. For small ε, these elementary motions will be localized in small neighborhoods of the vertices of the angles and, in the principal terms, will be determined by the principal terms of the expansion of w_1 near the angles. If, for the vertices of the angles lying at $x < 0$, the origin of the coordinate system x_1, y_1 is placed at the vertex of the angle, and the x_1-axis is directed along the wall in the direction of increasing φ_1, then the principal term of the expansion of w_1 will be $w_1 = f_1(t) z_1^n$, the opening of the angle is $(2n-1)\pi/n$; $f_1(t) = kv_0(t)$, $k = \text{const}$; $z_1 = x_1 + iy_1$. This reduces, for small ε, the consideration to case I with $f_2(t) = 0$.

Fig. 1

Cases I and II describe, in the main, the development of the second form of motion near the angular points of finite bodies. The resultant pressure force on the body in these cases is infinite because of the infinite extent of the bodies. To solve problems of flow past finite bodies with angles, one should take from the solutions of cases I, II the form and position of the lines γ and the distribution Γ along them, and, for computing the additional forces, use rule (A).

In the case considered by us, near each pair of vertices of angles A, B symmetric with respect to the y-axis, pieces γ_A, γ_B of vortex lines are formed. By formula (1), each pair γ_A, γ_B gives the additional force

$$Y_2 = 2\rho \frac{d}{dt} \left\{ \frac{1}{v_0(t)} \int_{\gamma_A} \psi_1 \, d\Gamma \right\}. \tag{2}$$

Equations (1) of work (^{1}) for $f_2(t) = 0$ are invariant with respect to the transformations $f_1'(t) = cf_1(t)$, $z' = c^{1/(2-n)}z$, $\Gamma' = c^{2/(2-n)}\Gamma$; $c = \text{const}$. Therefore, from equality (2) there follows the law of similarity (for small ε): for two similar laws of motion $v_0'(t)$, $v_0''(t)$, when $v_0'(t)/v_0''(t) = c = \text{const}$, the force Y_2 from a pair of symmetric vortex formations satisfies

$$Y_2'(t)/Y_2''(t) = c^{(2+n)/(2-n)}.$$

If the body has only one pair of symmetric angular points or several pairs of angular points with the same angle of opening, the law obtained is common to the whole body. If there are several pairs of such points with different opening angles, then the general law of similarity will be obtained as a linear combination of laws with different n. In the case of flow past a flat plate ($n = 1/2$), $Y_2'(t)/Y_2''(t) = c^{5/3}$.

a) $v_0(t) = k't^m$; $k' = \text{const}$, $m = \text{const}$. At each vertex where $x < 0$, the principal term of the expansion of w_1 will be $w_1 = k_1 t^m z_1^n$; $k_1 = k'k''$, $k'' = \text{const}$. For small ε, the problem reduces to case Ia) with $k_2 = 0$. Using formula (2), we obtain

$$\int_0^t Y_2(t)\,dt = 2\rho\,(k')^{(2+n)/(2-n)}(k'')^{4/(2-n)}\,c(n,m)\,t^{[\,n(m+1)+(2m+n)\,]/(2-n)}, \tag{3}$$

where $c(n, m)$ is a universal function of n, m for the self-similar motion of case Ia) ($k_2 = 0$):

$$c(n,m) = \int_\gamma R \sin\vartheta \, dG. \tag{4}$$

For a flat plate ($n = 1/2$) of width l, $k_1 = k'\sqrt{l}$, $k'' = \sqrt{l}$,

$$\int_0^t Y_2(t)\,dt = 2\rho\,(k')^{5/3} l^{4/3} c(1/2, m)\, t^{(2+5m)/3}. \tag{5}$$

If, for $t > 0$, the plate moves with constant velocity, then $m = 0$, $k' = v_0 = \text{const}$. Then

$$\int_0^t Y_2(t)\,dt = 2\rho l^{4/3} v_0^{5/3} c(1/2, 0)\, t^{2/3}. \tag{6}$$

Fig. 2

Figure 2: Fig. 2

b) $\omega_1 = \dfrac{n}{2i}\dfrac{v_0(t)l}{\cos\gamma}\left(z_2 - \dfrac{1}{z_2}\right)$, where

$$z_2 = \cos\gamma\left\{1 + \left(\frac{2z/l-1}{2z/l+1}\right)^n\right\} / \left\{1 - \left(\frac{2z/l-1}{2z/l+1}\right)^n\right\}, \tag{7}$$

$$n = 1/\{2 - (\beta_1+\beta_2)\}; \quad \gamma = \tfrac{1}{2}\pi(\beta_1-\beta_2)n; \quad \beta_1 = \text{const},\ \beta_2 = \text{const},$$

$$0 \leqslant \beta_1 + \beta_2 < 1.$$

We have the flow past a two-cornered body formed by arcs of circles; β_1, β_2 are the angles with the x-axis of the tangents at the point $z = -l/2$. At the point $z = -l/2$, ω_1 has the principal term of the expansion

$$\omega_1 = 2nv_0(t)l^{1-n}\cos\gamma\,(z+l/2)^n. \tag{8}$$

The problem reduces to case I with $f_2(t) = 0$, $f_1(t) = 2nv_0(t)l^{1-n}\cos\gamma$. Let $v_0(t) = k't^m$. The problem reduces to case Ia), where $k_1 = 2nk'l^{1-n}\cos\gamma$; in this case $k'' = 2nl^{1-n}\cos\gamma$. Equality (3) gives

$$\int_0^t Y_2(t)\,dt = 2\rho\,(k')^{(2+n)/(2-n)}\{2nl^{1-n}\cos\gamma\}^{4/(2-n)}c(n,m)\,t^{[n(m+1)+(2m+n)]/(2-n)}. \tag{9}$$

It follows from formula (9) that two bodies of the type considered, symmetric to one another with respect to the x-axis, experience the same force $Y_2(t)$, despite the absence of symmetry in the arrangement of γ (Fig. 2).

Fig. 2

c) The subcase of item b), when $2m + n = 0$. The investigation reduces to case IIb) with $k_2 = 0$. Formulas (9) of work (1) give $c(n, -n/2) = R_1\sin\vartheta_1\,G_1 = \pi\{(1-n)\sqrt{n}\sqrt{4n-1}\}^{2/(2-n)}$, and formula (9) gives

$$\int_0^t Y_2(t)\,dt = 2\rho\,(k')^{(2+n)/(2-n)}\{2n\cos\gamma\,l^{1-n}\}^{4/(2-n)}\pi\times$$

$$\times\{(1-n)\sqrt{n}\sqrt{4n-1}\}^{2/(2-n)}t^{n/2}. \tag{10}$$

Fig. 3

Figure 3: Fig. 3

For $\beta_1 = \beta_2,\ n = 1/2\ (m = -1/4)$ we obtain, for a plate of width l,

$$\int_0^t Y_2(t)\,dt = \frac{\pi}{2}\rho(k')^{5/3}l^{4/3}t^{1/4}. \tag{11}$$

IV. $w_1 = \dfrac{1}{4i}l(t)v_0(t)\left(z_2 - \dfrac{1}{z_2}\right);\quad z_2 = \left(1 + \sqrt{\dfrac{2z/l - 1}{2z/l + 1}}\right) \Big/ \left(1 - \sqrt{\dfrac{2z/l - 1}{2z/l + 1}}\right);$

$l(0) = 0$. This is the flow past a plate of variable width $l(t)$, arising from a point. The principal term of the expansion of w_1 near $z = -\frac{1}{2}l(t)$ will be

$$w_1 = v_0(t)\sqrt{l(t)}\sqrt{z + \tfrac{1}{2}l(t)}. \tag{12}$$

In the plane $z_1 = z + \frac{1}{2}l(t)$ the corresponding potential w_1 has the principal term of the expansion

$$w_1 = v_0(t)\sqrt{l(t)}\sqrt{z_1} + \frac{1}{2}\frac{dl(t)}{dt}z_1. \tag{13}$$

If $\dfrac{1}{l(t)}\displaystyle\int_0^t v_0(t)\,dt = \varepsilon \ll 1$, then the vortices are localized in neighborhoods, small in comparison with l, of the points $z = -l/2,\ z = l/2$, and the flow in the leading approximation reduces to case II, when

$$f_1(t) = v_0(t)\sqrt{l(t)}; \qquad f_2(t) = \tfrac{1}{2}\,dl(t)/dt$$

(Fig. 3).

Fig. 3

a) $l(t) = 4k_2t^{1/2}; \quad v_0(t) = \frac{1}{2}k_1k_2^{-1/2}t^{-1/2}; \quad k_1 = \text{const},\ k_2 = \text{const}.$

Here $\varepsilon = \frac{1}{16}k_1k_2^{-3/2}$. The problem reduces to item IIb) $(\beta = (16\varepsilon)^{-2/3})$. Setting, in the formulas (12) of work $^{(1)}$, $\vartheta_1 \to 0,\ \beta \to \infty$, using formula (3) and making the replacement

$$\int_\gamma R\sin\vartheta\,dG = R_1\sin\vartheta_1\,G_1,$$

we obtain

$$\int_0^t Y_2(t)\,dt = 4^{1/3}\pi\rho\, k_1^{5/3} k_2^{1/2}\sqrt{t}. \tag{14}$$

b) $l(t) = \dfrac{3}{m+1} k_2 t^{2/3(m+1)}$; $v_0(t) = \sqrt{(m+1)/3}\, k_1 k_2^{-1/2} t^{(2m-1)/3}$; $m =$ const,

$k_1 =$ const, $k_2 =$ const; $\varepsilon = \frac{1}{2}\sqrt{(m+1)/3}\, k_1 k_2^{-3/2}$. The problem reduces to item IIb) ($\varepsilon = \frac{1}{2}\sqrt{(m+1)/3}\,\beta^{-3/2}$). Using formula (1) and assuming valid the limiting equalities (14) of work (1), we obtain

$$\int_0^t Y_2(t)\,dt = \frac{\pi}{2}\rho\sqrt{3/(m+1)}(2m+1)^{-2/3} k_1^{5/3} k_2^{1/2} t^{2(m+1)/3}. \tag{15}$$

The main provisions of the work were reported by the author at the IX International Congress on Applied Mechanics in Brussels on 12 IX 1956.

Received
26 XI 1957

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Note: Figure translations are in progress. See original paper for figures.

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