PHYSICS

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1957

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Abstract

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PHYSICS

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ON THE THEORY OF DIFFUSION OSCILLATIONS IN GAS-DISCHARGE PLASMA

(Presented by Academician N. N. Bogolyubov, June 18, 1957)

Let us take, as usual ([1]), as the basis for the investigation of oscillations in a gas-discharge plasma a system of equations consisting of the diffusion equations for electrons and ions, Poisson' s equation for the electric field, and the energy equation for electrons. As an analysis of a more general system of equations has shown, owing to the strong heat exchange between ions and atoms of the neutral gas, temperature perturbations of the ions have a very weak effect on the character of the oscillations. Thus we have:

$$\frac{\partial n_1}{\partial t} - \frac{\partial (n_1 u_{\alpha_1})}{\partial x_\alpha} - D_1 \frac{\partial^2 n_1}{\partial x_\alpha^2} - D_T \frac{\partial^2 T_1}{\partial x_\alpha^2} = \beta n_1 - R; \tag{1}$$

$$\frac{\partial n_2}{\partial t} + \frac{\partial (n_2 u_{\alpha_2})}{\partial x_\alpha} - D_2 \frac{\partial^2 n_2}{\partial x_\alpha^2} = \beta n_1 - R, \tag{2}$$

$$\frac{\partial E_\alpha}{\partial x_\alpha} = 4\pi e\,(n_2 - n_1); \tag{3}$$

$$\frac{\partial}{\partial t}(n_1 c_{v1} T_1) + \frac{\partial}{\partial x_\alpha}(c_{v1} T_1 S_{\alpha_1}) =$$

$$= -eE_\alpha S_{\alpha_1} - (\beta n_1 - R)(\varepsilon_u + c_{v1} T_1) - \gamma_1 \frac{m}{M} c_{v1} \frac{T_1}{T_0}(T_1 - T_0) n_1. \tag{4}$$

Here the subscript 1 refers to electrons, the subscript 2 to ions; u_{α_1} and u_{α_2} are the drift velocities of electrons and ions, respectively; D_1 and D_2 are diffusion coefficients; D_T is the thermodiffusion coefficient for electrons; M is the mass of an atom of the neutral gas; T_1 is the electron temperature; $c_{v1} T_1$ is the internal energy of the electron gas; T_0 is the temperature of the neutral gas; γ_1 is the number of elastic collisions of electrons with atoms of the neutral gas (of the same order as the total number of collisions α_1); ε_u is the ionization energy; S_{α_1} denotes the electron flux due to the processes of diffusion, thermodiffusion, and drift in the electric field:

$$S_{\alpha_1} = -n_1 u_{\alpha_1} - D_1 \frac{\partial n_1}{\partial x_\alpha} - D_T \frac{\partial T_1}{\partial x_\alpha}. \tag{5}$$

The energy equation takes into account the transfer of energy with the electron flux S_α, the release of energy during the motion of electrons in the electric field, heat exchange with atoms of the neutral gas, and, finally, the expenditure of energy on the process of detaching an electron from an atom of the neutral gas and on imparting to these electrons an energy corresponding to the mean energy of electrons in the elementary volume.

In the central part of the tube the problem may be regarded as one-dimensional. Let us assume

$$n_1 = n_0 + \nu_1 e^{i(\omega t + kx)} \quad \nu_1 \ll n_0; \qquad n_2 = n_0 + \nu_2 e^{i(\omega t + kx)} \quad \nu_2 \ll n_0;$$
$$T_1 = T_{10} + \tau_1 e^{i(\omega t + kx)} \quad \tau_1 \ll T_{10}. \tag{6}$$

Substituting into (1)–(4), we obtain, as the condition for the existence of a nontrivial solution of the system, a dispersion equation of the form

$$\begin{aligned} D_T k^2 \Big[&(\beta + 4\pi e n_0 \mu_2)\, 2u_1 \frac{4\pi e n_0 \mu_1}{ik} \\ &- (i\omega + u_2 ik + D_2 k^2 + 4\pi e n_0 \mu_2) \left(2u_1 \frac{4\pi e n_0 \mu_1}{ik} - D_1 u_1 ik \right) \Big] + \\ &+ \frac{c_{v1} n_0}{m a_1} \left(i\omega + 2\gamma_1 \frac{m}{M} - \frac{c_{p1}}{c_{v1}} u_1 ik \right) \left[(i\omega - u_1 ik + D_1 k^2 + 4\pi e n_0 \mu_1 - \beta) \times \right. \\ &\left. \times (i\omega + u_2 ik + D_2 k^2 + 4\pi e n_0 \mu_2) - (\beta + 4\pi e n_0 \mu_2)\, 4\pi e n_0 \mu_1 \right] = 0. \end{aligned} \tag{7}$$

Hence, if it is assumed that the external electric field is not too large, namely:

$$u_1 / c_1 < \lambda_1 / l, \tag{8}$$

where u_1 is the drift velocity of electrons in the electric field; c_1 is their mean thermal velocity; λ_1 is the mean free path for electrons, and l is the wavelength of the process under study, $l = 2\pi/k$, we obtain a dispersion equation of a simpler type

$$\begin{aligned} (i\omega + D_1 k^2 - u_1 ik + 4\pi e n_0 \mu_1 - \beta)\,(i\omega + D_2 k^2 + u_2 ik + 4\pi e n_0 \mu_2) = \\ = (\beta + 4\pi e n_0 \mu_2)\, 4\pi e n_0 \mu_1. \end{aligned} \tag{9}$$

In this form it can be obtained without using the energy equation, which means that in this case the temperature perturbations of the electrons have only a weak influence on the character of the process.

In a weakly charged plasma, where the Debye radius d_1 is greater than or of the order of the wavelength l of the process under study, the coupling between electrons and ions is rather weak, and their independent motion is possible. Thus, under the condition $d_1 > l$, or $D_1/d_1^2 = 4\pi e n_0 \mu_1 \ll D_1 k^2 \simeq \omega \simeq \beta$, the right-hand side of equation (19) is negligibly small, and we obtain equations for the electron and ion waves

$$i\omega - u_1 ik + D_1 k^2 - \beta = 0; \tag{10}$$

$$i\omega + u_2 ik + D_2 k^2 = 0, \tag{11}$$

where the terms $u_1 k$ and $u_2 k$ are small in comparison with the diffusion terms, according to (8). Under the condition $k^2 = \beta/D_1$, an undamped electron wave is possible, propagating with the electron drift velocity u_1; in the remaining cases the dispersion equation (10) describes an electron wave with damping. The ion wave described by equation (11) practically does not exist because of very strong damping.

In plasmas where the concentration of charged particles is higher, electron–ion non-quasineutral waves are possible. Thus, for example, under the condition $l \simeq d_1\sqrt{\mu_1/\mu_2}$, from expression (9), using Thomson' s method ([5]), we obtain for the wave number of the ion wave the expression

$$k^2 = \frac{4\pi e n_0 \mu_2 + \beta}{D_1} + \frac{4\pi e n_0 \mu_1}{D_1} \frac{(\beta - i\omega)(4\pi e n_0 \mu_2 - i\omega)}{(4\pi e n_0 \mu_2)^2 - \omega^2}, \tag{12}$$

whose imaginary part is negative at low frequencies, when $\omega < 4\pi e n_0 \mu_2$; in this case we are dealing with damped waves. At frequencies of the order of $4\pi e n_0 \mu_2$ there is no clearly expressed wave, and the function k suffers a discontinuity. At higher frequencies, $\omega > 4\pi e n_0 \mu_2$, the imaginary part of expression (12) is positive, and we are dealing with a growing wave.

Finally, in real, sufficiently strongly charged plasmas, where the Debye radius is smaller than the period of the process,

$$d_1 < l, \tag{13}$$

we obtain the dispersion equation of quasi-neutral waves

$$\left(D_1 \frac{\mu_2}{\mu_1} + D_2\right) k^2 + (i\omega - \beta)\left(1 + \frac{\mu_2}{\mu_1}\right) = 0, \qquad |\gamma_1 - \gamma_2| \ll |\gamma_1|, \tag{14}$$

which we identify with striations ([2]), since the physical conditions under which striations are observed coincide with the conditions (8) and (13) for the applicability of equation (14). Encouraging results are obtained by comparing with experiment the theoretical dependences of the striation period on the tube radius and on the magnitude of the discharge current ([2]).

The velocity of moving striations is related to the magnitude of the spatial damping decrement* by the relation

$$v = \frac{\omega}{k} = -\left(D_1 \frac{\mu_2}{\mu_1} + D_2\right) \frac{2b}{1 + \mu_2/\mu_1}. \tag{15}$$

A. A. Zaitsev and Kh. A. Dzherpetov ([3]) pointed out the connection between the velocity of moving striations and the asymmetry of the discharge.

Simple particular cases can be obtained from the general dispersion equation (7) and under the condition of a very large external electric field $u_1/c_1 > \lambda_1/l$; however, in this case, owing to the substantial influence of electron temperature perturbations on the character of the process, it is necessary to take into account the dependence on the mean electron energy of such coefficients in the equations as the diffusion and thermodiffusion coefficients, the electron mobility μ_1, etc.

Let us take into account the dependence of the ionization coefficient on temperature by putting, according to Killian' s formula ([4]),

$$\beta = B\sqrt{\varkappa T_1} e^{-x_i} \left(1 + \frac{1}{x_i}\right),$$

where $\varkappa$ is Boltzmann' s constant, $x_i = \varepsilon_i/\varkappa T_1$.

Taking (6) into account, in the dispersion equation we must everywhere, instead of the constant ionization coefficient $\beta = \beta_0$, substitute

$$\beta = \beta_0 \left[1 + \frac{\tau_1}{T_{10}} \left({}^3/_2 + x_i + \frac{1}{x_i}\right)\right].$$

The dependence of the diffusion, thermodiffusion, and mobility coefficients of charged particles on the electron temperature is related to the dependence of the cross section for collisions of electrons with atoms of the neutral gas on the electron velocity and is different for different gases. Let the total number of electron collisions α_1 be proportional to the electron temperature T_1 to some power g. The general dispersion equation takes the form:

$$
\begin{aligned}
&\left(D_{Tk}^2 + gu_1 ik\frac{n_0}{T_{10}} - \beta_x\frac{n_0}{T_{10}}\right)\left[2u_1\frac{4\pi e n_0\mu_1}{ik}(\beta_0 + 4\pi e n_0\mu_2) - \right.\\
&\quad\left. -(i\omega + u_2 ik + D_2k^2 + 4\pi e n_0\mu_2)\left(2u_1\frac{4\pi e n_0\mu_1}{ik} - D_1u_1ik\right)\right] + \\
&+ \beta_x\frac{n_0}{T_{10}}\left[(i\omega + D_1k^2 + 4\pi e n_0\mu_1 - u_1ik - \beta_0)\,2u_1\frac{-4\pi e n_0\mu_1}{ik}\right.\\
&\quad\left. -4\pi e n_0\mu_1\left(u_1D_1ik - 2u_1\frac{4\pi e n_0\mu_1}{ik}\right)\right] + \\
&+ \left\{\frac{c_{v1}n_0}{m\alpha_1}\left[i\omega - \frac{c_{p1}}{c_{v1}}u_1ik + \beta_x + \beta_0 + 2\gamma_1\frac{m}{M}(1+g)\right]\right.\\
&\quad\left. +2u_1^2\frac{n_0}{T_{10}}g\right\}\left[(i\omega + D_1k^2 + 4\pi e n_0\mu_1 - u_1ik - \beta_0)(i\omega + D_2k^2 + \right.\\
&\quad\left. +4\pi e n_0\mu_2 + u_2ik) - 4\pi e n_0\mu_2(4\pi e n_0\mu_2 + \beta)\right] = 0,
\end{aligned}
\tag{16}
$$

where

$$
\beta_x = \beta_0\left[{}^3/_2 + x_i + \frac{1}{x_i} + (2g-1)\right].
$$

* The magnitude of the spatial damping decrement characterizes the degree of asymmetry of the distribution of charged particles in space: $n = n_0 + \nu e^{i(kx+\omega t)}e^{-bx}$.

In a strong external electric field and in a sufficiently strongly charged plasma (condition (13)), we obtain from expression (16) a dispersion equation which, for gases in which the thermodiffusion coefficient is nonzero and the mobility coefficient does not depend on temperature (which corresponds to a collision cross section inversely proportional to the electron velocity, as is observed, for example, in mercury), has the simple form and, in the stationary case,

$$
D_T\frac{T_{10}}{n_0}\frac{\mu_2}{\mu_1}k^2 = \beta_0\left(1 + \frac{\mu_2}{\mu_1}\right)\left(\frac{3}{2} + x_i + \frac{1}{x_i} + \lg -1\right) \tag{17}
$$

coincides in form with dispersion equation (14), if one takes into account that, for a thermodiffusion coefficient not equal to zero, the equality $D_1 = D_T T_{10}/n_0$ holds; thus, stationary strata are possible which obey the same laws as the strata described by equation (14), but are due to the process of thermodiffusion.

If the mobility coefficient depends on the electron temperature, then, irrespective of the character of thermodiffusion, in a large external electric field the wave is close in character to an ionic one:

$$i\omega + u_2 ik = 0 \tag{18}$$

and propagates with a velocity equal to the ion drift velocity.

Thus, analysis of the general dispersion equation of diffusion oscillations shows that in real plasmas, in which the Debye radius is of the order of hundredths or thousandths of a centimeter and, consequently, smaller than the period of the processes under study, quasineutral waves are observed.

In strong electric fields these oscillations are described by dispersion equations (17) and (18); in sufficiently weak fields (condition (8))—by equation (14). For an external field of intermediate magnitude, the dispersion equation is difficult to analyze directly because of the high algebraic powers with respect to the wave number.

In a very weakly charged plasma, where the Debye radius is of the order of or smaller than the period of the process under study, electron waves propagating independently are possible, as well as ion waves accompanied by the motion of electrons (12).

In conclusion I express my gratitude to Prof. M. F. Shirokov for recommending the topic and for valuable advice.

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Received
10 VI 1957

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Note: Figure translations are in progress. See original paper for figures.