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HYDROMECHANICS

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Abstract

Full Text

HYDROMECHANICS

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ON SOURCES OF ENERGY IN THE THERMAL EXCITATION OF SOUND

(Presented by Academician M. V. Keldysh, February 14, 1957)

1. The question of the source of energy in the thermal excitation of sound in a moving gas is treated incorrectly in a number of works ([1,2]). It is usually assumed that the only source of energy for maintaining sound oscillations is heat release. Since in many works the question of the source of energy is the starting point of the argument, all subsequent constructions and conclusions turn out to be erroneous to a considerable extent.

2. Let us write the equation for the energy flux

$$\mathcal{E} = \frac{\rho v^3}{2} + \rho v c_v T + pv; \tag{1}$$

here ρ is the density of the gas; p is the pressure; v is the flow velocity; c_{vT} is the internal energy.

The first two terms describe the transport of energy by the mass flux, and the last one its transfer by pressure. The excitation of acoustic oscillations is associated with the transfer of impulses by pressure, and therefore, of the three terms of $\mathcal{E}$, the last term pv is of principal interest.

Let a fixed heat-supply region Σ be bounded by two control planes normal to the axis of the one-dimensional flow under consideration. Suppose that the region Σ has a small extent in comparison with the wavelength of the excited oscillations, and therefore the hypothesis of stationarity can be applied to the processes taking place within Σ. Comparing the energy fluxes pv in the planes bounding the region Σ, let us pose the question of the amount of energy of this kind "radiated" by the heat-supply region, and of the source of this energy.

3. We shall denote by the indices 1 and 2 the quantities corresponding to the planes bounding Σ.

If one writes the law of conservation of energy separately for an element of the flow in a reference frame moving together with it, and separately for the center of gravity of this element, then, using the equation of continuity, it is not difficult to obtain the following equalities:

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$$Q = \rho v c_v (T_2 - T_1) + \int_\Sigma p\, dv; \tag{2}$$

$$P = \rho v \left(\frac{v_2^2}{2} - \frac{v_1^2}{2} \right) + \int_\Sigma v\, dp; \tag{3}$$

here Q is the heat and P the mechanical energy supplied to the gas in the region Σ. Obviously,

$$\mathcal{E}_2 - \mathcal{E}_1 = Q + P,$$

and

$$p_2 v_2 - p_1 v_1 = \int_\Sigma p\, dv + \int_\Sigma v\, dp. \tag{4}$$

Thus, the energy flux pv, "radiated" by the region Σ and equal to $p_2 v_2 - p_1 v_1$, consists of two terms. The first of them is associated with the supply of heat and the change in the internal-energy flux (2), and the second with the mechanical energy supplied and the change in the kinetic-energy flux (3).

It is interesting to note that specifying the flow parameters at the boundaries of the region Σ, although it determines the change in the fluxes pv, $\rho v c_v$, and $\rho \frac{v^3}{2}$, and consequently also $P + Q = \vartheta_2 - \vartheta_1$, is not sufficient to determine the part of the change in the flux pv that is associated with thermal quantities and the part associated with mechanical quantities. One must specify one more parameter, for example the heat supply Q. Specifying Q singles out an entire class of processes in the region Σ that are equivalent from this point of view; moreover, the actual law of variation of p and v along Σ proves to be immaterial.

4. Let the quantities p and v have harmonically time-varying components δp and δv, and let the period of these oscillations be τ. Then the mean acoustic energy radiated by the region Σ will be

$$A = \frac{1}{\tau} \int_0^\tau (\delta p_2 \delta v_2 - \delta p_1 \delta v_1)\, dt. \tag{5}$$

On the basis of equality (4) one may write

$$A = \frac{1}{\tau} \int_0^\tau \left[\iint_\Sigma \delta p\, d\delta v + \int_\Sigma \delta v\, d\delta p \right] dt. \tag{6}$$

The quantities in square brackets are no longer harmonic functions of time, although they continue to vary with period τ.

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Denoting by the symbol Δ the periodic, but not harmonic, components of order δ^2, we write, on the basis of equalities (2) and (3),

$$\int_0^\tau \left[\iint_\Sigma \delta p\, d\delta v\right] dt = \int_0^\tau \Delta\left[Q - \rho v c_v (T_2 - T_1)\right] dt; \tag{7}$$

$$\int_0^\tau \left[\int_\Sigma \delta v\, d\delta p\right] dt = \int_0^\tau \Delta\left[P - \rho v\left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right)\right] dt. \tag{8}$$

In the thermal excitation of sound, the mechanical energy supplied to the flow is $P = 0$. Therefore, according to equality (6):

$$A = \frac{1}{\tau}\int_0^\tau \left[\Delta Q - \Delta\rho v c_v (T_2 - T_1) - \Delta\rho v\left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right)\right] dt. \tag{9}$$

Formula (9) shows that the energy of acoustic oscillations may be borrowed both from the thermal terms (heat supply and internal energy) and from the kinetic-energy flux.

In this connection, two elementary processes are of particular interest. Let $\delta v_1 = \delta v = \delta v_2$. Then $d\delta v = 0$ and, according to formula (7), the first two terms in equality (9) give zero. This means that all the energy of the acoustic oscillations is borrowed from the kinetic energy of the flow. Conversely, in a process characterized by the condition $\delta p_1 = \delta p = \delta p_2$, all the energy of the acoustic oscillations will be borrowed from the thermal terms. In the first of the elementary processes named, a positive flux of acoustic energy A (5), i.e. excitation of the system, is obtained if the phase shift with respect to

between δv and $\delta X = \delta p_2 - \delta p_1$ will be less than $\pi/2$ in absolute value. Here δX has the physical meaning of the resistance of the zone Σ. In the second elementary process, excitation of the system is possible if the phase shift between δp and $\delta E = \delta v_2 - \delta v_1$ is less than $\pi/2$ in absolute value. Here δE characterizes the expansion in the region Σ. The latter statement is a generalization of Rayleigh' s excitation criterion to the case of a moving medium. If, under stationary conditions, the gas is motionless, then the phase of δE coincides with the phase of the heat supply ([3]), and this gives Rayleigh' s excitation criterion ([1]). The actual realization of both elementary processes is possible for the corresponding amplitudes and phases of the oscillatory component of the heat supply.

Usually both excitation mechanisms act simultaneously. Their interaction leads to a complex picture of the excitation conditions ([3]).

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CITED LITERATURE

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[2] G. H. Markstein, W. Squire, J. Acoust. Soc. Am., **27**, No. 3, 416 (1955).
[3] . . , DAN, **91**, No. 4, 749 (1953).

Note: Figure translations are in progress. See original paper for figures.

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