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Abstract

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Effective Cross Section for the "Stripping" of a μ-Meson from a Proton to a Deuteron

(Presented by Academician L. D. Landau, 13 VIII 1957)

The experimentally observed dependence of the catalysis of the nuclear reaction $p + D = \mathrm{He}^3$ by μ-mesons in hydrogen on the deuterium concentration is determined mainly by the process of stripping of the μ-meson from the proton to the deuteron ([1]). In the present work the effective cross section of this process is calculated.

The Hamiltonian of the $pD\mu$ system in mesoatomic units ($e = 1;\ h = 1;\ m_\mu = 1$) is

$$\hat{H} = -\frac{1}{2M_1}\Delta_{R_1} - \frac{1}{2M_2}\Delta_{R_2} - \frac{1}{2}\Delta_r - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R} \tag{1}$$

(M_1 and M_2 are the masses of the proton and deuteron; $R = |\mathbf{R}_1 - \mathbf{R}_2|$ is the distance between the nuclei; $r_1 = |\mathbf{r} - \mathbf{R}_1|$, $r_2 = |\mathbf{r} - \mathbf{R}_2|$ are the distances of the meson to the proton and deuteron).

Assuming that the transition of the μ-meson occurs between K-orbits, the wave function of the system may be written in the form

$$\Psi = A(R)\Sigma_g(\mathbf{r}, R) + B(R)\Sigma_u(\mathbf{r}, R); \tag{2}$$

Σ_g, Σ_u are the symmetric and antisymmetric wave functions of the μ-meson in the field of two fixed nuclei (depending on R as on a parameter)

$$\left(-\frac{1}{2}\Delta_r - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R}\right)\Sigma_{g,u} = E_{g,u}(R)\Sigma_{g,u}. \tag{3}$$

Substituting (2) into (1) and taking account of (3), we obtain, multiplying respectively by Σ_g and Σ_u and integrating over the coordinates of the μ-meson, the equations for $A(R)$ and $B(R)$:

$$-\frac{1}{2M}\Delta_R A + \left(E_g - \frac{1}{2M}K_{gg} - E\right)A - \frac{1}{2M}K_{gu}B - \frac{1}{M}S_{gu}\vec{\nabla}_R B = 0,$$

$$-\frac{1}{2M}\Delta_R B+\left(E_u-\frac{1}{2M}K_{uu}-E\right)B-\frac{1}{2M}K_{ug}A+\frac{1}{M}S_{gu}\vec{\nabla}_R A=0, \quad (4)$$

where

$$M=\frac{M_1M_2}{M_1+M_2};$$

$$\frac{1}{M}S_{gu}=\int\Sigma_g\left(\frac{1}{M_1}\vec{\nabla}_{R_1}+\frac{1}{M_2}\vec{\nabla}_{R_2}\right)\Sigma_u\,(d\mathbf{r}), \quad (5)$$

$$\frac{1}{M}K_{gg}=\int\Sigma_g\left(\frac{1}{M_1}\Delta_{R_1}+\frac{1}{M_2}\Delta_{R_2}\right)\Sigma_g\,(d\mathbf{r})\quad \text{etc.}$$

For $R\to\infty$

$$\Sigma_{g,u}=\frac{1}{\sqrt{2}}\left(\psi(r_1)\pm\psi(r_2)\right); \quad (6)$$

$\psi(r_1)$, $\psi(r_2)$ are the wave functions of the μ-meson at the proton and deuteron, respectively.

For the functions $a(R)=(A+B)\frac{1}{\sqrt{2}}$; $b(R)=(A-B)\frac{1}{\sqrt{2}}$, describing the motion of the deuteron relative to mesohydrogen and of the proton relative to mesodeuterium, we have the equations

$$\begin{gathered}
-\frac{1}{2M}\Delta_R a+\left[\frac{1}{2}(E_g+E_u)-\frac{1}{4M}(K_{gg}+K_{uu}+K_{gu}+K_{ug})-E\right]a+\\
+\left\{\frac{1}{2}(E_g-E_u)-\frac{1}{4M}(K_{gg}-K_{uu}+K_{ug}-K_{gu})\right\}b+\frac{1}{M}S\frac{db}{dR}=0,\\
-\frac{1}{2M}\Delta_R b+\left[\frac{1}{2}(E_g+E_u)-\frac{1}{4M}(K_{gg}+K_{uu}-K_{gu}-K_{ug})-E\right]b+\\
+\left[\frac{1}{2}(E_g-E_u)-\frac{1}{4M}(K_{gg}-K_{uu}-K_{ug}+K_{gu})\right]a+\frac{1}{M}S\frac{da}{dR}=0,\\
S=(S_{gu})_R.
\end{gathered} \quad (7)$$

It can be shown that, to within exponentially small terms and terms $\sim\frac{1}{R^4}$ for $R\gg 1$,

$$E_g+E_u\simeq -1,$$

$$K_{gg} + K_{uu} \simeq 2 \int \psi(r_1) \Delta_{r_1} \psi(r_1)\,(dr) = -1$$

$$K_{gu} + K_{ug} \simeq 2 \frac{M_2 - M_1}{M_2 + M_1} \int \psi(r_1) \Delta_{r_1} \psi(r_1)\,(dr) = -\frac{M_2 - M_1}{M_2 + M_1}.$$

The quantities $K_{gg} - K_{uu}$, $K_{gu} - K_{ug}$, S are exponentially small, and the corresponding terms in (7), containing $\frac{1}{M}$, at distances R essential for the problem under consideration, may be neglected in comparison with $E_g - E_u$.

In approximation (6), $E_g - E_u \simeq {}^4/_3 R e^{-R}$; a more accurate calculation (taking into account the distortion of the μ-meson wave function near one nucleus by the action of the other) gives

$$E_g - E_u \simeq \frac{4}{e} R e^{-R}.$$

Assuming that the collision occurs at thermal velocities, in the relative motion of the nuclei it is sufficient to take into account only the S-wave. The equations for the radial functions $u = Ra(R)$ and $v = Rb(R)$ take the form

$$\begin{gathered} \frac{d^2u}{dR^2} + 2M\varepsilon u + M(E_u - E_g)v = 0, \\ \frac{d^2v}{dR^2} + 2M\left[\varepsilon + \frac{1}{2}\left(\frac{1}{M_1} - \frac{1}{M_2}\right)\right]v + M(E_u - E_g)u = 0. \end{gathered} \tag{8}$$

(The energy of the mesoatom of hydrogen has been chosen as the zero of energy.) The term $\frac{1}{2M_1} - \frac{1}{2M_2}$ takes into account the difference between the binding energies of the μ-meson at the proton and the deuteron.

$M(E_u - E_g)$, in the range of distances essential for the problem under consideration, can be approximated by the function $e^{-\alpha(R-R_1)}$ with $R_1 = 3.38$; $\alpha = 1 - \frac{1}{R_1} \simeq 0.704$. In this same region the energy ε may be neglected.

Equations (8) then take the form ($x = R - R_1$):

$$\frac{d^2u}{dx^2} + e^{-\alpha x}v = 0, \qquad \frac{d^2v}{dx^2} + k_0^2 v + e^{-\alpha x}u = 0; \tag{9}$$

$$k_0^2 = -\frac{M_2 - M_1}{M_2 + M_1} \simeq \frac{1}{3}.$$

As $x \to -\infty$ ($R_0^2 \ll e^{-\alpha x}$), the solutions of equations (9) are

$$u + v = C_1 \left[J_0 \left(\frac{2}{\alpha} e^{-\alpha x/2} \right) + \delta N_0 \left(\frac{2}{\alpha} e^{-\alpha x/2} \right) \right],$$

$$u - v = C_2 K_0 \left(\frac{2}{\alpha} e^{-\alpha x/2} \right), \tag{10}$$

where $J_0(z), N_0(z)$ are Bessel functions of the first and second kinds; $K_0(z)$ is a Bessel function of imaginary argument; C_1, C_2, δ are constants.

The $pD\mu$ system has a level with energy very close to 0 ([2]). As a result, a resonance occurs, and the constant δ is negligibly small. The constants C_1, C_2 are determined from the condition that the asymptotic form of the solution as $x \to +\infty$ has the form

$$u \simeq x + \beta; \qquad v \simeq \gamma e^{ik_0 x}.$$

The quantities γ and β are found by numerical integration of system (9) with the initial conditions (10)*.

For the solutions

$$u_1 = v_1 = J_0 \left(\frac{2}{\alpha} e^{-\alpha x/2} \right)$$

and

$$u_2 = -v_2 = K_0 \left(\frac{2}{\alpha} e^{-\alpha x/2} \right),$$

taken at $x = -2$, the asymptotic form as $x \to +\infty$, respectively, is

$$u_1 \simeq a_1 x + b_1, \qquad v_1 \simeq c_1 \cos(k_0 x + \delta_1),$$

$$a_1 = -0.489, \qquad b_1 = 1.257, \qquad c_1 = 1.06, \qquad \delta_1 = 258^\circ;$$

$$u_2 \simeq a_2 x + b_2, \qquad v_2 \simeq c_2 \cos(k_0 x + \delta_2),$$

$$a_2 = 0.202, \qquad b_2 = -0.281, \qquad c_2 = 0.254, \qquad \delta_2 = 29^\circ.$$

The effective cross section of "transfer" is

$$\sigma = 4\pi\frac{k_0}{k}|\gamma|^2 = 4\pi\frac{k_0}{k}\frac{\sin^2(\delta_1-\delta_2)}{\left(\frac{a_1}{c_1}\right)^2+\left(\frac{a_2}{c_2}\right)^2-2\frac{a_1}{c_1}\frac{a_2}{c_2}\cos(\delta_1-\delta_2)} \simeq 1.5\pi\frac{k_0}{k}.$$

(Between the numbers $a_1, a_2, b_1, b_2, \ldots$ there is the relation

$$a_1 b_2 - b_1 a_2 = k_0 c_1 c_2 \sin(\delta_1-\delta_2),$$

by virtue of which $\operatorname{Im}\beta = k_0|\gamma|^2$.)

The value obtained agrees with that adopted to explain the experimental data in work ([1]),

$$\sigma = \pi\frac{k_0}{k}$$

(the best agreement is obtained for

$$\sigma = 0.5\pi\frac{k_0}{k}$$

).

The value of the effective cross section given in ([3]), calculated in the Born approximation ([4]) (which is essentially the opposite limiting case), differs from ours by a factor of $1.5\cdot10^2$. It must also be noted that in Jackson' s paper ([3]), because nonorthogonal wave functions were used in the calculations (Appendix D), the probability of mesomolecule formation is overestimated by three orders of magnitude in comparison with that calculated in ([2]). In this connection, in ([3]) an incorrect conclusion was drawn that the dependence of the catalysis of the pD reaction on the concentration D is determined by competition between the processes of "jumping" of the μ-meson from the proton to the deuteron and the processes of formation of the mesomolecule $pp\mu$. As shown in ([1]), the indicated dependence is determined by the probability of the "jump" of the μ-meson to the deuteron during the lifetime of the μ-meson with respect to the decay $\mu \to e + \nu + \tilde{\nu}$.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.