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K. N. FEDOROV

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Abstract

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GEOPHYSICS

K. N. FEDOROV

ON ANNUAL AND SEMIANNUAL OSCILLATIONS OF THE GENERAL CIRCULATION OF THE OCEANS

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Judging from the data of Ichie ([1]) and Masuzawa ([2]) for 1940 and 1952–1953, the transport of the Kuroshio has two maxima per year: the principal one in autumn and a somewhat smaller one in spring. Harmonic analysis showed that the semiannual wave is predominant in amplitude. Approximately the same is characteristic of the transport of the Gulf Stream in 1937–1939, with the difference that here the maxima occur in summer and winter ([3]). The data on the velocity of the North Equatorial Current in the Atlantic are contradictory. Harmonic analysis of Fuglister' s data ([4]) reveals a noticeable predominance of the semiannual wave. Galle' s data ([4]) correspond to an almost pure oscillation with an annual period. We emphasize that for the Kuroshio as well the predominance of the semiannual amplitude is much less pronounced in the annual course of the **velocity** than in the annual course of the **transport**.

If the predominance of the semiannual wave were also characteristic of the general atmospheric circulation over the oceans, this would fully explain the facts cited, and the correctness of the figures from ([5]) could be doubted. However, harmonic analysis of the mean wind stress, calculated from monthly maps of atmospheric pressure ([6,7]) for the entire region of the North Pacific trade winds and for the entire belt of westerly winds, showed that its oscillations are rather annual in character, while the semiannual component in them is comparatively small. The same, to a somewhat lesser degree, is also characteristic of the Atlantic, as is confirmed by analysis of wind speed from ([4,5]). It is possible that the predominance of the semiannual amplitude in the Kuroshio is a consequence of the monsoons. But this is only a partial explanation, since in the oscillations of the Gulf Stream transport, for example, the semiannual wave predominates despite the fact that monsoons in this region are weakly expressed. Evidently, such a ratio of amplitudes should be regarded as a characteristic feature not only of the currents mentioned, but in general of both anticyclonic gyres of the waters of the northern parts of the oceans, and its causes must have the character of a large-scale climatic phenomenon.

Such phenomena, in addition to the annual course of wind speed, include the displacement of the center of atmospheric circulation (the North Pacific maximum of atmospheric pressure in the Pacific Ocean and the Azores maximum in the Atlantic) in the meridional direction with a period of 1 year. For the North Pacific maximum, for example, the extreme northern position (in July) is 40° N, while the extreme southern position (in January) is about 25° N. Therefore the trade-wind belt is approximately 1600 km wider in summer than in winter; and if the shape of the Pacific Ocean is taken into account, it turns out that the area occupied by the trade winds exceeds the area of the westerly-wind belt in winter by 2–3 times, and in summer by 6–7 times. It is clear that the trade winds, their nonuniformity and variability, play the principal role in forming the main features of the circulation of the waters of the northern parts of the oceans, and that the periodic change in the area of the trade winds and the associated oscillation in the amount of motion imparted to the water by the trade winds cannot but

affect the intensity of water circulation in the anticyclonic gyre.

Aiselin ([3]), for example, believes that the winter maximum of the Gulf Stream transport is due to the maximum intensity of the anticyclonic wind system over the North Atlantic in winter, and the summer maximum to the northward displacement of the Azores maximum and, consequently, of the North Equatorial Current, a large part of which, as a result, feeds the Gulf Stream while bypassing the Caribbean Sea and the Florida Strait. This is correct. The point, however, is not that in this case, as Aiselin writes, "the friction concentrated in the southwestern region decreases." The sum of two harmonic oscillations with an annual period but with different phases (the oscillation associated with the winter maximum of the trade-wind speed, and the oscillation due to the summer "minimum of friction") would, by the law of superposition of harmonic oscillations, also be an annual wave, only with some resultant phase. The cause lies in oscillations of the amount of motion imparted by the trade winds to the water. If one provisionally takes as a measure of the amount of motion the product of the mean speed of the trade winds by the width of their belt, bearing in mind that both these quantities are harmonic functions of the annual period, then this measure will be a harmonic function of semiannual frequency. This reasoning does not embrace all the complexity of the interaction of the oscillating fields of wind and currents, but it helps one understand, in the most general terms, the nature of the semiannual wave in the intensity of water circulation in the northern parts of the oceans. In turn, it is clear why the transport of a current, which depends not only on the current speed but also on its width, has a larger amplitude of semiannual oscillations than the speed, which depends mainly on the intensity of the wind stress.

Let us illustrate all that has been said by a simple theoretical scheme. We shall find the water transport in a mean transverse section of a closed rectangular region, strongly elongated in the zonal direction. Place the origin of coordinates at the lower boundary of the baroclinic layer; direct the z-axis vertically upward, the x-axis westward along the southern boundary of the region, coinciding with the equator, and the y-axis northward along the section. Integrating the equations of motion from 0 to the sea surface (H) and denoting the components of the integral motion along the x and y axes by S_x and S_y, respectively, we, neglecting the quantities S_y and $\frac{\partial^2 S_x}{\partial x^2}$, obtain the following equation for the total flow along the x-axis:

$$\frac{\partial S_x}{\partial t} = A_l \frac{\partial^2 S_x}{\partial y^2} + f(y,t), \tag{1}$$

where A_l is the coefficient of lateral turbulent friction, t is time, and

$$f(y,t) = \tau_x(y,t) - \frac{\partial P}{\partial x},$$

where, in turn, $\tau_x(y,t)$ is the tangential wind stress and

$$P = \int_0^H p\,dz$$

(p is pressure).

The solution of (1), satisfying zero boundary and initial conditions, is known ([8]). We shall examine it for a wind distribution describing, in the most general terms, the meridional variation of the wind in the trade-wind region—westerly winds:

$$\tau_x(y,t) = [\tau_0 + \tau_1 \cos(\sigma t - \varphi_1) + \tau_2(2\sigma t - \varphi_2)] \left[\left(k_1 - \frac{y}{l}\right) + k^2 \cos(\sigma t - \varphi_3)\right]. \tag{2}$$

Here σ is the frequency of the annual wave; φ_1, φ_2, and φ_3 are, respectively, the phases of the annual and semiannual waves and of the displacement of the center of the circulation. Positive values of τ_x in (2) correspond to the trade winds, and negative values of τ_x to westerly—

winds, the tangential stress of both pulsates with annual and semiannual frequencies, and the boundary between them periodically shifts in the meridional direction about the mean position k_1 with amplitude k_2.

Neglecting in the periodic part of (2) terms of small amplitude and of frequency higher than semiannual, we obtain for the periodic part S_x the expression:

$$S_x(y,t) = \sum_{n=1}^{\infty} \sin\frac{\pi n}{l} y \Bigg\{ \frac{2\tau_1}{\pi n\sqrt{A_n^2+\sigma^2}} [\cos(\sigma t - \varphi_1 - \alpha_1) -$$

$$- e^{-A_n t}\cos(-\varphi_1 - \alpha_1)] \left[(-1)^n + 2k_1\left(\frac{1-\cos\pi n}{2}\right)\right] +$$

$$+ \frac{4\tau_0 k_2}{\pi n\sqrt{A_n^2+\sigma^2}} [\cos(\sigma t - \varphi_3 - \alpha_1) - e^{-A_n t}\cos(-\varphi_3 - \alpha_1)] \left(\frac{1-\cos\pi n}{2}\right) -$$

$$- \frac{4\Delta P_1}{\pi n\sqrt{A_n^2+\sigma^2}} [\cos(\sigma t - \beta_1 - \alpha_1) - e^{-A_n t}\cos(-\beta_1 - \alpha_1)] \left(\frac{1-\cos\pi n}{2}\right) +$$

$$+ \frac{2\tau_2}{\pi n\sqrt{A_n^2+4\sigma^2}} [\cos(2\sigma t - \varphi_2 - \alpha_2) -$$

$$- e^{-A_n t}\cos(-\varphi_2 - \alpha_2)] \left[(-1)^n + 2k_1\left(\frac{1-\cos\pi n}{2}\right)\right] +$$

$$+ \frac{2\tau_1 k_2}{\pi n\sqrt{A_n^2+4\sigma^2}} [\cos(2\sigma t - \varphi_3 - \varphi_2 - \alpha_2) -$$

$$- e^{-A_n t}\cos(-\varphi_3 - \varphi_2 - \alpha_2)] \left(\frac{1-\cos\pi n}{2}\right) -$$

$$- \frac{4\Delta P_2}{\pi n\sqrt{A_n^2+4\sigma^2}} [\cos(2\sigma t - \beta_2 - \alpha_2) - e^{-A_n t}\cos(-\beta_2 - \alpha_2)] \left(\frac{1-\cos\pi n}{2}\right) \Bigg\}, \tag{3}$$

$$n = 1,\ 2,\ 3,\ 4, \dots,$$

where $A_n = \left(\frac{\pi n}{l}\right)^2 A_1$, $\operatorname{tg}\alpha_1 = \frac{\sigma}{A_n}$, $\operatorname{tg}\alpha_2 = \frac{2\sigma}{A_n}$, and ΔP_1, ΔP_2 and β_1, β_2 are the amplitudes and phases of the annual and semiannual waves in the total fluctuation of the integral pressure gradient $\frac{\partial P}{\partial x}$ of surge origin.

A regular oscillatory process is established at sufficiently large t, when all terms containing $e^{-A_n t}$ vanish. Under these conditions we find the discharge Q in the transverse section by integrating (3) with respect to y from 0 to the northern boundary of the region l:

$$Q(t) = \sum_{n=1}^{\infty} \{Q_1(n)\cos[\sigma t - \varepsilon_1(n)] + Q_2(n)\cos[2\sigma t - \varepsilon_2(n)]\} \tag{4}$$

$$n = 1,\ 3,\ 5,\ 7,\ 9, \dots$$

Here in $Q_1(n) = \frac{4l_1 q}{\pi^2 n^2\sqrt{A_n^2+\sigma^2}}$ and $Q_2(n) = \frac{4lq_2}{\pi^2 n^2\sqrt{A_n^2+4\sigma^2}}$, $\varepsilon_1(n)$ and

$\varepsilon_2(n)$ combine the resultant amplitudes and phases of the annual and semiannual waves.

The phases β_1 and β_2 are found from the continuity equation $\dfrac{\partial S_x}{\partial x} = \dfrac{\partial \zeta}{\partial t}$, assuming that P is proportional to the excess of level ζ. In this case the phases β_1 and β_2 must exceed by 90° the corresponding phases of the discharge oscillations in a purely drift current. The amplitudes ΔP_1 and ΔP_2 are found from the condition of equality of the amplitudes of the corresponding waves of the gradient and drift discharges, which is equivalent to the natural requirement of equality of the volumes of water transported over a half-period in opposite directions by the drift and gradient currents.

Let us assign the numerical values: $\tau_0 = 0.70$ dyn/cm^2; $\tau_1 = 0.50$ dyn/cm^2 and $\tau_2 = 0.11$ dyn/cm^2; $\varphi_1 = 360°$; $\varphi_2 = 270°$; $\varphi_3 = 180°$; $k_1 = 0.6$; $k_2 = 0.1$; $\sigma = 2 \cdot 10^{-7}$ s^{-1} and $l = 5.56 \cdot 10^8$ cm. These figures define a field of tangential stress such that the mean stress over the whole trade-wind belt, computed from (2), reproduces, to within ±10%, the actual mean stress obtained from (6). The results of the calculation for $Al = 10^{10}$ CGS and $n = 1$ are as follows:

α_1	α_2	$An \cdot 10^6$	q_1	$\varepsilon_1(n)$	q_2	$\varepsilon_2(n)$	$Q_1(n)$(cm^3/sec$\cdot 10^{-14}$)	$Q_2(n)$
32°	52°	0.32	0.068	184°	0.101	236°	0.41	0.45

The series (4) converges very rapidly, since the amplitudes of the transports $Q_1(n)$ and $Q_2(n)$ decrease in proportion to n^4, so that in practice the term with $n = 3$ may already be neglected.

The order of magnitude of the quantities obtained is quite plausible. The amplitudes of the oscillations amount to 10-20% of the stationary transport, which is close to the value of 15% for the North Atlantic (9). The semiannual amplitude predominates, though not as strongly as is obtained from analysis of the observational results. The latter circumstance may be attributed to the unaccounted-for influence of the monsoons. Let us note that, if the phase φ_1 were not 360°, but 180° or 156°, as in (5), then $Q_1(n)$ would considerably exceed $Q_2(n)$. Evidently, this was precisely what was characteristic, on average, for the series of years to which the data of (5) correspond. The moment of maximum transport in the section, for each wave, coincides with the maximum transport of the North Equatorial Current and with the minimum transport of the current directed eastward, whose phases should therefore differ from ε_1 and ε_2 by 180°. In turn, the phases at the western and eastern ends of the region should differ from ε_1 and ε_2, respectively, by 90° and 270°. Thus, for example, from our scheme for the phase of the annual wave in the transport of the Kuroshio we obtain 184° + 90° = 274°, which agrees satisfactorily with the values 227, 313, and 303° obtained by harmonic analysis of the data (1,2). The phase of the semiannual wave cannot be compared with the observational-analysis data, since the latter are highly contradictory, although most of them differ from $\varepsilon_2 + 90^\circ$ by more than 90°.

Institute of Oceanology
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.