Estimation of the Conditional Coefficient of Thermal Diffusivity in Modeling Atmospheric Processes

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Abstract

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Estimation of the Conditional Coefficient of Thermal Diffusivity in Modeling Atmospheric Processes

(Presented by Academician V. V. Shuleikin, 10 IV 1957)

As early as 1921, Defant ([1]) proposed calculating heat transfer in the atmosphere as the product of a certain conditional coefficient of thermal conductivity and the mean value of the projection of the temperature gradient onto the horizontal plane (we shall call this projection simply the temperature gradient).

On the terrestrial globe and on models, the gradient of the mean temperature of the atmosphere (or of the medium modeling it) has no sharp discontinuities, just as the mean heat flux, for example, from south to north has none. The mechanism of heat transfer may change from one region to another. Thus, in middle latitudes macroturbulence associated with cyclogenesis predominates, while in low latitudes the trade-wind and monsoon transports are more strongly expressed. As may be concluded from experiments on models, the character of the process is apparently determined by the "thermal head," i.e., by the magnitude of the heat flux itself and by the properties of the medium ([2], p. 432).

Using the concept of a conditional coefficient of thermal conductivity, Ångström ([3]) obtained a distribution of mean zonal temperatures over the terrestrial globe close to the observed one.

Modeling atmospheric circulation in a rotating flat vessel with a circular heater at the center, Scaife ([4]) came to the conclusion that the temperature distribution with distance from the center is close to that given by the theory of thermal conductivity around an infinite rectilinear heat source. In this case the coefficient of thermal diffusivity should be taken as considerably larger than the molecular one. In his experiments this coefficient proved to be equal to $a_k^2 = 4.1 \cdot 10^{-2}$ cm^2/sec, instead of $a_m^2 = 1.33 \cdot 10^{-3}$ cm^2/sec.

For the case of the atmosphere, V. V. Shuleikin assumed that the apparent coefficient of thermal diffusivity over a circular island is directly proportional to the square of the radius of the island (([5]), p. 390). The coefficient of proportionality has the dimension of inverse time; however, its physical meaning is not disclosed.

In modeling, an estimate of the apparent coefficient of thermal diffusivity is needed in order to substitute it into the corresponding similarity criteria. Setting ourselves only the task of obtaining the order of magnitude of the desired quantity, we shall use the following rough scheme for the reasoning.

Let us first suppose that, over the model of a circular heated island of radius R, there occurs an impulsive replacement n times per second of warm fluid by cold fluid from the surrounding region. Let the thickness of the fluid layer be H, the density ρ, and the heat capacity c. The heating of each portion occurs by an amount $\Delta\overline{T}$, equal to the difference of the mean temperatures of the "atmosphere" over the "island" and over the sea. Then, per unit time, from the "island" there will be removed

the amount of heat equal to

$$Q_1 = \pi n R^2 \rho c H \Delta\overline{T}. \tag{1}$$

On the other hand, assuming that heat transfer above the shoreline is described by the heat-conduction equation with a fictitious coefficient of thermal diffusivity a_ϕ^2, the total heat flux may be written in the form

$$2\pi R a_\phi^2 \rho c H \left.\frac{dT}{dr}\right|_{r=R} = Q_2. \tag{2}$$

Equating expressions (1) and (2), we find the desired fictitious coefficient of thermal diffusivity in the form

$$a_\phi^2 = \frac{nR\Delta\overline{T}}{2\left.\dfrac{dT}{dr}\right|_R}. \tag{3}$$

The order of magnitude of the temperature gradient above the shoreline is determined by the degree of heating $\Delta\overline{T}$ and by the dimensions of the island R. Introducing the proportionality coefficient χ, one may write

$$\left.\frac{dT}{dr}\right|_R = \chi\frac{\Delta\overline{T}}{R}. \tag{4}$$

The numerical value of χ is determined by the law of temperature distribution above the island.

Instead of the frequency of changes n, one may introduce the period between changes

$$\tau = \frac{1}{n}. \tag{5}$$

Then, instead of (3), we obtain

$$a_\phi^2 = \frac{R^2}{2\tau\chi}. \tag{6}$$

If we abandon the initial hypothesis of periodic changes of the air masses above the island, and assume that uniform closed circulation with mean radial velocity u_r ensures, over the time $R/u_r = \tau$, a complete replacement of the air above the island, where it is heated by the same amount $\Delta\overline{T}$, then the expression for the coefficient of apparent thermal diffusivity will have the form

$$a_\phi^2 = \frac{Ru_r}{2\chi}. \tag{7}$$

In this case, each particle of air or of the liquid modeling it will gradually, before replacement, be heated to the maximum temperature, and the coefficient χ, apparently, will be close to unity.

Let us apply formula (1) to the results obtained by Scorer ([4]). Put $\chi = 1$, and the maximum velocity $u_r = 5 \cdot 10^{-2}$ cm/sec. The mean velocity, for an almost sinusoidal law of its variation, represented, for example, in Fig. 20 of work ([4]), will be of the order $\overline{u}_r = 3 \cdot 10^{-2}$ cm/sec. The radius of the heated region of the model was $R = 3$ cm. Consequently, by formula (7) we obtain

$$a_\phi^2 = \frac{3 \cdot 3 \cdot 10^{-2}}{2} = 4.5 \cdot 10^{-2} \text{ cm}^2/\text{sec}$$

instead of the value $a_\phi^2 = 4.1 \cdot 10^{-2}$ cm/sec, obtained by Scorer. The agreement may be considered satisfactory.

Let us now apply the formula obtained to terrestrial conditions. For Australia one may take $R = 10^8$ cm. The wind speed, normal to the shoreline, is of the order of 5 m/sec. Then

$$a_{\text{Av}}^2 \approx \frac{10^8 \cdot 5 \cdot 10^2}{2} = 2.5 \cdot 10^{10} \text{ cm}^2/\text{sec}.$$

If, for the continent of Eurasia, one takes $R = 4.1 \cdot 10^8$ cm, as V. V. Shuleikin does, then for the same mean wind velocities we obtain

$$a_{\text{Eur}}^2 \simeq \frac{4.1 \cdot 5 \cdot 10^{10}}{2} \simeq 10^{11} \text{ cm}^2/\text{sec}.$$

The estimates given are consistent with the values of the coefficient of kinematic viscosity of the atmosphere obtained in dynamical meteorology when allowing for macroturbulence of the type of cyclonic formations ([1], [3], [5], [6]).

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Note: Figure translations are in progress. See original paper for figures.