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Abstract

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PHYSICS

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THEORY OF THE INTERACTION OF EVAPORATING OR GROWING DROPS AT LARGE DISTANCES

An isothermal treatment of the problem of the motion of a drop in a diffusion field (1) made it possible to establish the attractive or repulsive forces acting between fog droplets or between a droplet and an extended moist surface, and exerting a significant influence on their motion relative to the medium. However, in this treatment the diffusion forces in the first approximation turned out to be compensated by the influence of the Stefan flow, so that the velocity of motion of the drop relative to the phase-transition surface was found to be zero. In the present work, however, it will be shown that when account is taken of the heat-transfer process occurring in parallel with diffusion, such compensation in the first approximation is, in the general case, violated; in connection with this, a substantial influence of diffusion forces on the deposition and coagulation of an aerosol is possible.

1. To solve the problem posed, we shall use the simplifications discussed in (1). We shall confine ourselves to the most important case of a small partial density of vapor, which makes it possible to use the linearized system of hydrodynamic equations for the mixture. We shall calculate the interaction of drops at a distance so large that each of them may be regarded as being in prescribed homogeneous fields of diffusion and temperature, caused by the phase transition at the surface of the other drop. All processes will be regarded as quasi-stationary, since the time of thermal and diffusion relaxation of drops of several microns in size, for which the influence of the forces considered is most noticeable, is completely negligible. In order to obtain the boundary conditions on the surface of a drop with allowance for the heat of condensation being released, it is necessary to consider the temperature fields outside the drop $T_1(r_2, \theta_2)$ and inside it $T_2(r_2, \theta_2)$, which, under the adopted simplifications, satisfy Laplace's equation. Taking the foregoing into account, using the boundary conditions for the velocity field discussed in (1), and introducing a spherical coordinate system whose origin is placed at the center of drop 2 of radius R_2, while the axis ($\theta_2 = 0$) is directed toward increasing vapor concentration, we obtain the following system of equations and boundary conditions:

$$\operatorname{grad} p = \eta \Delta \mathbf{v}, \qquad \operatorname{div} \mathbf{v} = 0; \tag{1}$$

$$\Delta \rho' = 0, \qquad \Delta T_1 = 0, \qquad \Delta T_2 = 0; \tag{2}$$

$$v_\theta(R_2, \theta_2) = 0, \qquad \rho'' v_r(R_2, \theta_2) + D \frac{\partial \rho'}{\partial r_2}(R_2, \theta_2) = 0; \tag{3}$$

$$\mathbf{v}\big|_{r \to \infty} = -D \frac{\nabla \rho'_{\text{vn}}}{\rho''}; \tag{4}$$

$$\rho'\big|_{r \to \infty} = \rho'_\infty + A r_2 \cos \theta_2 \left(A = \frac{\partial \rho'}{\partial (r_2 \cos \theta_2)} \bigg|_{\substack{\theta_2 = \pm 90^\circ \\ r_2 \to \infty}} \right),$$

$$T_1\big|_{r \to \infty} = T_\infty + B r_2 \cos \theta_2 \left(B = \frac{\partial T}{\partial (r_2 \cos \theta_2)} \bigg|_{\substack{\theta_2 = \pm 90^\circ \\ r_2 \to \infty}} \right); \tag{5}$$

$$T_1(R_2, \theta_2) = T_2(R_2, \theta_2); \tag{6}$$

$$\rho'(R_2, \theta_2) = \rho'_s[T_1(R_2, \theta_2)] \simeq \rho'_s(T^*)[1 + (T_1(R_2, \theta_2) - T^*)/\Omega]; \tag{7}$$

$$LD \frac{\partial \rho'}{\partial r_2}(R_2, \theta_2) = -k_1 \frac{\partial T_1}{\partial r_2}(R_2, \theta_2) + k_2 \frac{\partial T_2}{\partial r_2}(R_2, \theta_2), \tag{8}$$

where p is the pressure; $\mathbf{v}$ is the velocity of the particles of the mixture; ρ' is the partial density of vapor; ρ'' is the partial density of air; η is the viscosity coefficient of air; D is the coefficient of diffusion of vapor in air; L is the specific heat of vaporization; k_1 and k_2 are the thermal conductivities of air and water; $\rho'_s(T)$ is the density of saturated vapor at temperature T. Boundary condition (7) takes into account that the amount of heat q released on the surface of the drop is partly conducted outward (q_1), partly into the drop (q_2), where, naturally, $q = q_1 + q_2$. Expressing the dependence of the density of saturated water vapor on temperature by Magnus' empirical formula $\rho'_s(T) = C \exp\left(\frac{aT}{b+T}\right)$, where $a = 17.2$, $b = 235$, and taking into account that in fogs the deviation of the drop temperature from the psychrometric temperature T^* is small and usually satisfies the condition $T - T^* \ll \Omega = (b + T^*)/a$, we have simplified (7) by expanding $\rho'_s(T)$ in a series about $T = T^*$, retaining only the first two terms.*

2. Solving the system of equations (2) together with the boundary conditions (5)–(8), we obtain

$$\rho'(r_2,\theta_2) = \rho'_\infty + (\rho'_s(T^*) - \rho'_\infty)\frac{R_2}{r_2} + Ar_2\cos\theta_2 -$$

$$-\left(A - 3\,\frac{LDA + k_1B}{k_2 + 2k_1 + 2LD\rho'_s(T^*)/\Omega}\,\frac{\rho'_s(T^*)}{\Omega}\right)\frac{R_2^3}{r_2^2}\cos\theta_2; \qquad (9)$$

$$T_1(r_2,\theta_2) = T_\infty + (T^* - T_\infty)\frac{R_2}{r_2} + Br_2\cos\theta_2 -$$

$$-\left(B - 3\,\frac{LDB + k_1B}{k_2 + 2k_1 + 2LD\rho'_s(T^*)/\Omega}\right)\frac{R_2^3}{r_2^2}\cos\theta_2; \qquad (10)$$

$$T_2(r_2,\theta_2) = T^* + 3\,\frac{LDA + k_1B}{k_2 + 2k_1 + 2LD\rho'_s(T^*)/\Omega}\,r_2\cos\theta_2. \qquad (11)$$

As is seen from (11), an additional polarization temperature is superposed on the mean temperature of the drop, which coincides with the psychrometric temperature. In a homogeneous external temperature and diffusion field the drop undergoes diffusion-thermal polarization.

3. Using (9) and (3), we find

$$v_r(R_2,\theta_2) = u\cos\theta_2 + V, \qquad (12)$$

where

$$u = \frac{3D}{\rho''}\left(A - 2\,\frac{LDA + k_1B}{k_2 + 2k_1 + 2LD\rho'_s(T^*)/\Omega}\,\frac{\rho'_s(T^*)}{\Omega}\right), \qquad (13)$$

$$V = D[\rho'_s(T^*) - \rho'_\infty]/R\rho''.$$

The obtained boundary condition (12), which takes diffusion-thermal polarization into account, differs from the corresponding boundary condition for purely diffusional polarization (1) by the quantity of the coefficient u_*, which, for the particular case of diffusional polarization, can be obtained from (13) by

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* As is known, the psychrometric temperature for the case of evaporation is determined from the condition of complete compensation of the heat absorbed in evaporation, $LD(\rho'_s(T^*) - \rho'_\infty) = k(T_\infty - T^*)$. This term is conveniently applied to the temperature determined from this condition also in condensation.

setting its second term equal to zero. Therefore the velocity field caused by diffusion-thermal polarization, $v_1(r_2, \theta_2)$, satisfying (3) and the zero boundary condition at infinity, can be obtained by substituting (13) into the formulas derived for the case of diffusional polarization, and the corresponding diffusion-thermal polarization force F_1 through the diffusional-polarization force calculated in (1), by substituting (13) in the form

$$F_1 = \frac{6\pi\eta D R_2}{\rho''}\left(A - 2\frac{LDA + k_1 B}{k_2 + 2k_1 + 2LD\rho_s'(T^*)/\Omega}\,\frac{\rho_s'(T^*)}{\Omega}\right). \qquad (14)$$

The total velocity field satisfying the system of equations (1) and the boundary conditions (3), (4) is expressed by the superposition of $v_1(r_2, \theta_2)$ and the Stokes velocity field $v_2(r_2, \theta_2)$, caused by the external Stefan flow, satisfying (4) and the zero boundary condition on the surface of the drop. Correspondingly, the total diffusion force is expressed by the sum of (14) and the force caused by the external Stefan flux, calculated by Stokes' formula $F_2 = -6\pi\eta R_2 DA/\rho''$.

$$F = F_1 + F_2 = -2\,\frac{12\pi\eta D R_2 \rho_s'(T^*)\,(LD\nabla\rho \;+ k_1\nabla T\;)}{(k_2 + 2k_1 + 2LD\rho_s'(T^*)/\Omega)\,\Omega\rho''}, \qquad (15)$$

where, taking (5) into account, we have passed to vector form.

The sign of the diffusion force is determined by the expression in parentheses, which represents the resultant transfer of heat to the surface by means of molecular heat conduction and vapor diffusion. When heat is released, the surface repels drops, and when heat is absorbed, it attracts them.

4. When a surface is flowed around by a turbulent stream, the diffusion force is localized in a thin boundary layer of thickness δ and, according to (15), is expressed as follows:

$$F = \frac{12\pi\eta D R_2 \rho_s'(T^*)}{\Omega\rho''\delta\,(k_2 + 2k_1 + 2LD\rho_s'(T^*)/\Omega)}\left[LD\left(\rho_\infty' - \rho_s'(T)\right) + k_1(T_\infty - T)\right], \qquad (16)$$

where T is the surface temperature. A surface whose temperature is below the psychrometric temperature attracts drops, while at a temperature above the psychrometric temperature it repels them. For example, when fog moves along a cooled coil, diffusion forces promote the deposition of drops. Diffusion forces should hinder the icing of surfaces in supercooled clouds, since the freezing of drops on them is accompanied by the release of heat. Since in this case a turbulent flow regime usually occurs—for example, during the icing of an aircraft or of wires in a strong wind—the contribution of diffusion forces to this important effect can be estimated with the aid of (16); moreover, the force thus calculated for micron-size drops at a temperature difference of several degrees exceeds the force of gravity hundreds of times when $\delta \sim 100\,\mu$.

5. From drop 1 of radius R_1, on drop 2 of radius R_2, located at a distance $r_{12} \gg R_1, R_2$, there acts, according to (15), the diffusion force

$$\mathbf{F}_{12} = \frac{12\pi\eta D R_1 R_2 \rho'_s(T^*)}{\Omega\rho''\left(k_2 + 2k_1 + 2LD\rho'_s(T^*)/\Omega\right)}\left[LD\left(\rho'_s[T_1(R_1)] - \rho'_\infty\right) + k_1\left(T_1(R_1) - T_\infty\right)\right]\frac{\mathbf{r}_{12}}{r_{12}^3}. \tag{17}$$

The force exerted by droplet 2 on droplet 1 is obtained from (17) by interchanging the indices.

A special consideration shows that the temperature of the droplets at any distance between them coincides with the psychrometric temperature if no heat is absorbed or released in the droplets, so that the force of diffusional interaction in this important case is equal to zero.

Solar radiation hinders coagulation processes in clouds and fogs, since the temperature of droplets, upon absorption of radiant energy, is established at a level higher than the psychrometric temperature. The temperature of radioactive droplets is higher than the psychrometric temperature owing to the heat released in radioactive decay, so that radioactive fogs should have enhanced stability.

A freezing droplet, releasing heat, should be a center of repulsion for other droplets, while a melting hailstone, absorbing heat, should be a center of attraction.

Taking into account the identical character of the dependence of diffusional and electrical forces on distance, it is not difficult, by substituting in the formula obtained by L. M. Levin[2] for the capture efficiency of smaller fog droplets by a falling droplet (due to electrical forces) the quantity corresponding, in the case of diffusional interaction, to the product of the electrical charges, to obtain an expression for the capture efficiency of droplets by a falling melting hailstone:

$$E = \frac{36\pi\eta D\rho'_s(T^*)\left[LD\left(\rho'_\infty - \rho'_s[T_1(R_1)]\right) + k_1\left(T_\infty - T_1(R_1)\right)\right]}{\Omega\rho'_v\rho_{\mathrm{B}}g\left(k_2 + 2k_1 + 2LD\rho'_s(T^*)/\Omega\right)R_1^3\left(1 - R_2^2/R_1^2\right)}, \tag{18}$$

where ρ_{B} is the density of water, and g is the acceleration of gravity.

A calculation performed by this formula shows that the growth of cloud elements by coalescence in the range $2 \div 15\,\mu$ can occur not only owing to electrical forces[2], but also owing to diffusional forces.

During bubbling, the efficiency of aerosol absorption decreases as the temperature rises and becomes equal to zero at the boiling temperature[3], which apparently is explained by diffusional repulsion of the particles from the surface of the bubble, which has a higher temperature than the air.

The influence of diffusional forces on the deposition of dust in the respiratory tract deserves attention, since, for the small diameter of the lower respiratory tract, the diffusional forces here should be very large. The temperature of the walls of the respiratory tract is apparently higher than the psychrometric temperature, so that the forces considered should facilitate the protection of the lungs from dust.

It may thus be stated that the long-range forces between aerosol particles during their evaporation or condensation, considered here for the first time, whose importance for consideration in aerosol mechanics is obvious, are of considerable applied interest for the study and solution of a number of aerosol problems relating to various branches of science and technology.

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Note: Figure translations are in progress. See original paper for figures.