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Abstract

Full Text

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Diffraction of Sound by a Thin Finite Elastic Cylindrical Shell

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1. We study the diffraction of a plane sound wave

$$p_i = \exp[ik\cos\theta\cos\varphi + ik\sin\theta z]$$

(θ is the angle of incidence) by a thin* finite cylindrical shell of circular cross section, using the method of an integro-differential equation ([1,2]) and under more general assumptions than in the recently published works ([3,4]), where only the special case of normal incidence of a plane wave on an infinite shell was considered. It is assumed that the shell, whose axis coincides with the z-axis of the cylindrical coordinate system r, φ, z, is hinged at the points $z = 0, d$ into a cylindrical absolutely rigid and immovable screen, and that the vibrations of the shell are described by equations ([5]).

2. Using Green' s theorem ([6]), the radiation condition ([7]), and the condition of equality of the normal velocities at the boundary of the shell with the surrounding medium, the solution of the wave equation $(\Delta + k^2)p = 0$ can be represented in the form

$$p(r,\varphi,z) = p_i(r,\varphi,z) + p_r(r,\varphi,z) -$$

$$-i\omega\rho\int_s G(r,a,\varphi-\varphi',z-z')w(\varphi',z')\,ds'. \quad (1)$$

Here s is the surface of the shell; $p_r(r,\varphi,z)$ is the known part of the solution, describing the scattering field of an absolutely rigid infinite cylinder ([8]), and $G(r,a,\varphi-\varphi',z-z')$, $r > r' = a$, is the Green' s function which is a solution of the equation

$$(\Delta + k^2)G(r,r',\varphi-\varphi',z-z') = -\frac{\delta(r-r')}{r}\delta(\varphi-\varphi')\delta(z-z')$$

for $\operatorname{Im} k > 0$, bounded everywhere except at the point r', φ', z', and satisfying on the surface of the cylinder the condition $\partial G/\partial n = 0$;

$$G(r, a, \varphi - \varphi', z - z') = \frac{1}{4\pi^2 a} \int_{-\infty}^{+\infty} \sum_{m=0}^{\infty} \varepsilon_m \frac{H_m^{(1)}(r\sqrt{k^2 - \xi^2})}{\sqrt{k^2 - \xi^2} H_m^{(1)}(a\sqrt{k^2 - \xi^2})} \times$$

$$\times \cos m(\varphi - \varphi') \exp[i\xi(z - z')]\, d\xi; \qquad r > a. \tag{2}$$

By substituting (1) into equations ([5]), the problem is reduced to solving a system of integro-differential equations for $w(\varphi, z)$

$$\omega^2 \rho_1 u + E_1 \left[\frac{\partial^2 u}{\partial z^2} + \frac{1}{2a}(1 - \nu) \frac{\partial^2 u}{\partial \varphi^2} + \frac{1}{2a}(1 + \nu) \frac{\partial^2 v}{\partial \varphi \partial z} + \frac{\nu}{a} \frac{\partial w}{\partial z} \right] =$$

$$= \frac{i\omega}{2} \frac{\nu}{1 - \nu} \frac{\partial p(a, \varphi, z)}{\partial z}; \tag{3}$$

* A shell is meant whose thickness is much smaller than the length of the longitudinal wave in the material of the shell.

$$\omega^2 \rho_1 v + E_1 \left[\frac{1}{a^2} \frac{\partial^2 v}{\partial \varphi^2} + \frac{1}{2}(1 - \nu) \frac{\partial^2 v}{\partial z^2} + \frac{1}{2a}(1 + \nu) \frac{\partial^2 u}{\partial \varphi\, \partial z} + \frac{1}{a^2} \frac{\partial w}{\partial \varphi} \right] +$$

$$+ \frac{1}{8} h^2 E_1 \frac{\nu}{1 - \nu} \left(\frac{1}{a^4} \frac{\partial^3 w}{\partial \varphi^3} + \frac{1}{a^4} \frac{\partial w}{\partial \varphi} \right) = \frac{i\omega}{2a} \frac{\nu}{1 - \nu} \frac{\partial p(a, \varphi, z)}{\partial \varphi}; \tag{4}$$

$$\omega^2 \rho_1 w - E_1 \left(\frac{\nu}{a} \frac{\partial u}{\partial z} + \frac{1}{a^2} \frac{\partial v}{\partial \varphi} + \frac{w}{a^2} \right) -$$

$$- \frac{h^2}{24} \frac{E_1}{1 - \nu} \left[2(1 - \nu) \left(\frac{\partial^4 w}{\partial z^4} + \frac{2}{a^2} \frac{\partial^4 w}{\partial z^2 \partial \varphi^2} + \frac{1}{a^4} \frac{\partial^4 w}{\partial \varphi^4} \right) + \right.$$

$$\left. + (4 - \nu) \frac{1}{a^4} \frac{\partial^2 w}{\partial \varphi^2} + (2 + \nu) \frac{w}{a^4} \right] = i\omega \left[\frac{1}{h} + \frac{1 - 2\nu}{2(1 - \nu)a} \right] p(a, \varphi, z), \tag{5}$$

where

$$p(a, \varphi, z) = p_i(a, \varphi, z) + p_r(a, \varphi, z) -$$

$$- \frac{i\omega\rho}{4\pi^2} \int_0^{2\pi} \int_0^{d} \int_{-\infty}^{+\infty} \sum_{m=0}^{\infty} \varepsilon_m w(\varphi', z') \frac{H_m^{(1)}\left(a\sqrt{k^2 - \xi^2}\right)}{\sqrt{k^2 - \xi^2}\, H_m'^{(1)}\left(a\sqrt{k^2 - \xi^2}\right)} \times$$

$$\times \cos m(\varphi - \varphi') \exp[i\xi(z - z')]\, d\varphi'\, dz'\, d\xi. \tag{5a}$$

After determining $w(\varphi, z)$ from equations (3), (4), (5) and substituting the value $w(\varphi, z)$ into (1), we obtain the required solution $p(r, \varphi, z)$. In expressions (1)–(5): p is the sound pressure; c is the speed of sound in the medium; ρ is the density of the medium; a is the radius; h is the shell thickness; $E_1 = \dfrac{E}{1-\nu^2}$ is Young's modulus; ν is Poisson's ratio; ρ_1 is the density of the shell material; n is the inward normal to the shell surface; u, v, w are the velocities of displacement of the shell surface in the axial, circumferential, and radial directions; $H_n^{(1)}(\alpha)$ is a Hankel function of the first kind. A prime denotes differentiation with respect to the argument; $\varepsilon_0 = 1$, $\varepsilon_m = 2$ for $m = 1, 2, 3, \ldots$.

3. The solution of the system of integro-differential equations (3), (4), (5) can be reduced to the solution of linear algebraic equations if the displacement velocities $u(\varphi, z)$, $v(\varphi, z)$, $w(\varphi, z)$ and the pressures $p_i(a, \varphi, z) + p_r(a, \varphi, z)$ are expanded in series in eigenfunctions satisfying the homogeneous differential equations of the shell and the boundary conditions at the points $z = 0, d$ ([9]). In particular,

$$w(\varphi, z) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{mn} \cos m\varphi \, \sin \frac{\pi n z}{d}. \tag{6}$$

Determining approximately* the coefficients a_{mn} from the solution of the algebraic equations, substituting the values a_{mn} into (6), and then into (1), and using the method of steepest descent in the integration, we finally obtain

$$\begin{aligned} p(r, \varphi, z) \simeq p_i(r, \varphi, z) + p_r(r, \varphi, z) + \frac{\rho c^2 d}{2\pi^2 \omega a \cos\theta \cos\theta'} \frac{\exp[ikR]}{R} \times \\ \times \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \varepsilon_m^2 \exp\left[-i \frac{m-3}{4} \pi\right] \frac{\cos m\varphi}{H_m'^{(1)}(ka\cos\theta)\, H_m'^{(1)}(ka\cos\theta')\, (Z_{mn} + Z_{mnmn})} \times \\ \times \left[\frac{\exp[i(k\sin\theta\, d + \pi n)] - 1}{k\sin\theta\, d + \pi n} - \frac{\exp[i(k\sin\theta\, d - \pi n)] - 1}{k\sin\theta\, d - \pi n} \right] \left[\frac{\exp[i(k\sin\theta'\, d + \pi n)] - 1}{k\sin\theta'\, d + \pi n} - \right. \\ \left. - \frac{\exp[i(k\sin\theta'\, d - \pi n)] - 1}{k\sin\theta'\, d - \pi n} \right]. \end{aligned} \tag{7}$$

* The approximation consists, in essence, in the assumption that the system of orthogonal eigenfunctions satisfying the homogeneous differential equations of the shell and the chosen boundary conditions satisfies the inhomogeneous differential (integro-differential) equations of the shell.

Calculations show that in a number of important cases this assumption is justified and is confirmed experimentally. For details, see, for example, ([1,2,10]).

Here $R = \sqrt{r^2 + z^2}$; θ' is the scattering angle;

$$Z_{mnmn} = \frac{i\omega\rho}{2\pi d} \int_0^d \int_0^d \int_{-\infty}^{+\infty} \varepsilon_m \frac{H_m^{(1)}\left(a\sqrt{k^2-\xi^2}\right)}{\sqrt{k^2-\xi^2}\, H_m'^{(1)}\left(a\sqrt{k^2-\xi^2}\right)} \sin\frac{\pi n z}{d} \times$$

$$\times \sin\frac{\pi n z'}{d} \exp[i\xi(z-z')]\; dz\, dz'\, d\xi; \qquad (8)$$

$$Z_{mn} = +i\,\frac{E_1 h}{\omega a^2}\frac{D}{D_1}; \qquad (9)$$

$$D_1 = \begin{vmatrix} -\frac{\pi n h}{2d}\frac{\nu}{1-\nu}; & \frac{1+\nu}{2}\, m\,\frac{\pi n a}{d}; & \frac{\rho_1\omega^2 a^2}{E_1} - \frac{\pi^2 n^2 a^2}{d^2} - \frac{1-\nu}{2} m^2 \\ \frac{mh}{2a}\frac{\nu}{1-\nu}; & \frac{\rho_1\omega^2 a^2}{E_1} - m^2 + \frac{1-\nu}{2}\frac{\pi^2 n^2 a^2}{d^2}; & \frac{1+\nu}{2}\, m\,\frac{\pi n a}{d} \\ -\left[1+\frac{1-2\nu}{2(1-\nu)}\frac{h}{a}\right]; & -m; & \nu\frac{\pi n a}{d} \end{vmatrix}; \qquad (10)$$

$$D = \begin{vmatrix} \nu\frac{\pi n}{d}a; & \frac{1+\nu}{2}\, m\pi n\,\frac{a}{d}; & \frac{\rho_1\omega^2 a^2}{E_1} - \frac{\pi^2 n}{d} \\ -m + \frac{1}{8}\frac{h^2}{a^2}\frac{\nu}{1-\nu}m(m^2-1); & \frac{\rho_1\omega^2 a^2}{E_1} - m^2 + \frac{1-\nu}{2}\frac{\pi^2 n^2 a^2}{d^2}; & \frac{1+\nu}{2} \\ \frac{\rho_1\omega^2 a^2}{E_1} - 1 - \frac{h^2 a^2}{24}\frac{1}{1-\nu}\,[2(1-\nu)\times \\ \times\left(\frac{\pi^4 n^4}{d^4} + 2\frac{m^2}{d^2}\frac{\pi^2 n^2}{a^2} + \frac{m^4}{a^4}\right) - (4-\nu)\frac{m^2}{a^4} + \frac{2+\nu}{a^4}\Big]; & -m; & \nu\frac{\pi}{} \end{vmatrix} \qquad (11)$$

Z_{mnmn} and Z_{mn} are, respectively, the radiation impedance and the mechanical impedance of the natural mode of oscillation of the shell with number mn. If the frequency of the incident wave coincides with one of the natural frequencies of the shell, $Z_{mn} + \operatorname{Im} Z_{mnmn} = 0$, and under the condition $|\pi n| = |k \sin\theta\, d| = |k \sin\theta'\, d|$ significant scattering is observed not only in the specular direction, but also in the direction opposite to that of the incident wave (nonspecular reflection) ([1,11]). It can be shown that nonspecular reflection is observed whenever the phase velocity of the incident wave along the shell coincides with the velocity of one of the normal—free—waves in the shell propagating along its axis ([11]).

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Note: Figure translations are in progress. See original paper for figures.