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Abstract

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THE INFLUENCE OF A SMALL BLUNTING OF THE LEADING EDGE OF AN AIRFOIL ON ITS FLOW AT HIGH SUPERSONIC SPEED

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In supersonic flow past airfoils or bodies of revolution, a small blunting of the front end may be the cause of a noticeable change in the flow in a region whose extent is many times greater than the characteristic size of the blunting. At moderate supersonic speeds, immediately behind the blunting there is observed a region of reduced pressure, caused by the expansion flow along a strongly curved surface; if the Reynolds number, computed from the transverse size of the blunting, is sufficiently large, then the flow past the curved portion is accompanied by local separation of the boundary layer ([1]). At very high supersonic speeds, a region of increased pressure extends a considerable distance downstream from the blunting; this effect is caused by the action of viscosity near the sharp leading edge ([2]).

When considering the flow in a region whose extent is large in comparison with the characteristic size of the blunting, one may neglect the presence of the blunting itself, i.e., consider the flow past a sharpened body, and replace the action of the blunting on the flow by a concentrated force applied at the front point of the body.

Restricting ourselves in what follows to the case of flow past a symmetric airfoil, we shall consider the flow only in the upper half-plane. When the airfoil moves in a stationary gas with velocity V, the resultant of the forces exerted by the upper half of the blunting on the gas may be resolved into a component X in the direction of motion and a component Y in the perpendicular direction. The component X, equal in magnitude to one half of the drag of the blunting, performs work, increasing the energy of the gas. The energy of the gas contained in a layer of unit width perpendicular to the flow direction increases, as a result of the action of the blunting, by $X \cdot l$. The component Y does no work, but, like the component X, imparts momentum to the gas. The momentum imparted by the blunting to the gas in a layer of unit width in the direction perpendicular to the direction of flight is equal to Y/V. It should be noted that, in the case of flow past a blunting with a detached shock wave, the value of this momentum must include the momentum of the pressure forces acting on the gas from the portion of the plane of symmetry between the detached shock wave and the leading edge of the body.

Let us estimate the influence of blunting on the flow past a thin airfoil by a flow of high supersonic speed. It is known ([3]) that the problem of steady flow past bodies in this case is equivalent to the problem of one-dimensional unsteady motion with plane waves, arising when a gas is displaced by a piston moving with velocity $U = V \operatorname{tg} \alpha$ (α is the angle of inclination of an element of the surface of the body being flowed past to the direction of the oncoming flow; the coordinate x along the flow must then be replaced by Vt, t being time).

For a thin body blunted in front, the equivalent problem of unsteady gas motion consists in the following. At the initial moment,

at time zero an energy E (per unit area) is released on the plane, and an impulse I normal to it is imparted to the gas above the plane; at the same moment the plane begins to move with velocity U into the region occupied by the gas. If $E = 0$, $I = 0$, and $U = \text{const}$, then the resulting motion is self-similar and the problem has a simple exact solution. In cases where $E \neq 0$, $I = 0$, $U = 0$, or $E = 0$, $I = 0$, $U = Ct^n$, and the resulting shock wave has very high intensity, the motion is also self-similar and exact solutions are known for the corresponding problems ([4,5]). In the latter case the motion will be self-similar also for $E \neq 0$, if $n = -1/3$. If $n \neq -1/3$, the motion is not self-similar, and an exact solution can be obtained only by numerical methods, similarly to how, for example, the solution of the point-explosion problem with counterpressure taken into account was obtained ([6]). An approximate analytical solution of these problems can be obtained by the method previously applied by the author to the explosion problem ([7]).

We shall use, to solve the formulated equivalent problem of one-dimensional motion, an approximate method based on the fact that in the case of strong shock waves the principal mass of the gas in the disturbed region is concentrated in a thin layer behind the shock wave. We shall assume that the entire mass of the gas in the disturbed region moves together with the shock wave (this assumption is the more accurate, the greater the intensity of the shock wave and the closer to unity the ratio of specific heats γ). For simplicity we restrict ourselves to the case $U = \text{const}$, which corresponds to flow past a wedge. Neglecting the initial pressure of the gas (the inclusion of which causes no additional difficulties), from the law of conservation of energy we obtain

$$\rho_1 R \cdot \frac{\dot{R}^2}{2} + (R - Ut)\frac{p}{\gamma - 1} = E + U \int_0^t p\, dt. \tag{1}$$

The momentum equation gives

$$\rho_1 R\dot{R} = I + \int_0^t p\,dt. \tag{2}$$

Here $R(t)$ is the law of motion of the shock wave; $p(t)$ is the pressure in the region between the shock wave and the piston; ρ_1 is the initial density of the gas; a dot denotes differentiation with respect to time.

Eliminating the pressure p from equations (1) and (2), we obtain

$$\rho_1 R\frac{\dot{R}^2}{2} + \frac{1}{\gamma-1}(R-Ut)\frac{d}{dt}\rho_1 R\dot{R} = E - IU + \rho_1 R\dot{R}U. \tag{3}$$

Leaving aside the case $U = 0$, which was considered by the author earlier in the formulation presented here ([8]), we introduce for measuring length the scale

$$L = \frac{E - IU}{\rho_1 U^2}$$

and for measuring time the scale L/U. Equation (3) then takes the form

$$\frac{1}{\gamma-1}(R-t)\frac{d}{dt}R\dot{R} = 1 + R\dot{R} - \frac{1}{2}R\dot{R}^2. \tag{4}$$

This equation has a unique solution $R^*(t)$, satisfying the condition $R(0) = 0$ and existing for all $t \geq 0$. For small values of t,

$$R^* = \left(\frac{9}{2}\frac{\gamma-1}{\gamma}\right)^{1/3} t^{2/3}.$$

For large t, the solution R^* tends to the asymptote

$$R = \frac{2\gamma}{\gamma+1}\,t + \frac{\gamma^2-1}{2\gamma}$$

(which is an exact solution of equation (4)).

Since, as $t \to 0$, $R^*\dot{R}^* = O(t^{1/8})$, the solution $R^*(t)$ corresponds to the case $I = 0$.

For small values of the half-angle α at the vertex of the wedge, the quantity $IU = Y\,\mathrm{tg}\,\alpha = \frac{1}{4}c_y\rho_1 V^2 h\,\mathrm{tg}\,\alpha$ is small in comparison with the quantity $E = X = \frac{1}{4}c_x\rho_1 V^2 h$ (c_y, c_x are of order unity, h is the transverse dimension of the blunting). This makes it possible to use the solution $R^*(t)$ to estimate the influence of blunting of the leading edge of a thin wedge on its flow in a stream of very large supersonic velocity.

Fig. 1

Figure 1: Fig. 1

Fig. 1

The shape of the shock wave is determined by the formula

$$\frac{R\,\mathrm{tg}^2\,\alpha}{X/(\rho_1 V^2)} = R^*\left(\frac{x\,\mathrm{tg}^3\,\alpha}{X/(\rho_1 V^2)}\right).$$

For the pressure distribution over the wedge we obtain

$$\frac{p}{\rho_1 V^2\,\mathrm{tg}^2\,\alpha} = \dot{W}\left(\frac{x\,\mathrm{tg}^3\,\alpha}{X/(\rho_1 V^2)}\right)$$

$$(W = R^*\dot{R}^*).$$

Graphs of these dependences are given in Fig. 1.

The case $\alpha = 0$ corresponds to the flow past a flat plate with a blunted leading edge. Using the asymptotic expressions for the functions R^* and $\dot{W}$, for this case we find

$$R = \left(\frac{9}{2}\frac{\gamma-1}{\gamma}\frac{X}{\rho_1 V^2}\right)^{1/3} x^{2/3}, \qquad \frac{p}{\rho_1 V^2} = \frac{2}{9}\left(\frac{9}{2}\frac{\gamma-1}{\gamma}\frac{X}{\rho_1 V^2}\right)^{2/3} x^{-2/3}.$$

For $\gamma = 1.4$, these formulas are close to the exact solution of the problem of a strong explosion ([4]), which agrees satisfactorily with the experimental results on the flow past a blunted plate at large supersonic velocity ([9]).

Fig. 1 shows that blunting of the leading edge causes a substantial change in the flow pattern and in the pressure distribution over the wedge being flowed around. For thinner wedges the change is stronger than for thicker ones.

Let us calculate the drag $\bar{X}$ of a wedge of length l with a blunted leading edge:

$$\bar{X} = X + \int_0^l p\,\mathrm{tg}\,\alpha\,dx = X + XW\left(\frac{l\,\mathrm{tg}^3\,\alpha}{X/(\rho_1 V^2)}\right).$$

The increase in the drag of the blunted wedge in comparison with the drag of the sharp wedge is expressed by the formula

$$\bar{X} - \bar{X}_{\chi=0} = \left[1 + W\left(\frac{l \operatorname{tg}^3 \alpha}{X/(\rho_1 V^2)}\right) - \left(\frac{2\gamma}{\gamma+1}\right)^2 \frac{l \operatorname{tg}^3 \alpha}{X/(\rho_1 V^2)}\right] X = nX.$$

The quantity n varies from unity to γ as $\dfrac{l \operatorname{tg}^3 \alpha}{X/(\rho_1 V^2)}$ varies from 0 to ∞. Thus, the change in pressure in the flow caused by blunting of the edge leads to an increase in the drag of the wedge by an amount greater than the drag of the blunting itself. In the case of thin wedges, for which the main part of the drag is caused by blunting, this additional increase in drag may be substantial.

In conclusion we give data on the accuracy of the approximate solution obtained for the problem of one-dimensional unsteady flow, by comparing it with the exact solution in two limiting cases. For $E = 0$, $U \gg a_1$ (a_1 is the speed of sound in the undisturbed gas) and $\gamma = 1.4$, the exact solution gives $R^* = 1.20\,t$, while the approximate one gives $R^* \simeq 1.17\,t$; for $E \neq 0$, $U = 0$, $a_1 = 0$, the exact solution gives $R^* = 1.23\,t^{2/3}$, while the approximate one gives $R^* \simeq 1.09\,t^{2/3}$.

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Note: Figure translations are in progress. See original paper for figures.