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Abstract

Full Text

Geophysics

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Dependence of the Turbulence Coefficient on the Parameters of Sea Waves

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The value of the turbulence coefficient is needed in solving a number of problems in the dynamics and thermics of the sea. The study of the mechanism of vertical exchange in the sea is also of great interest for marine hydrobiology.

1. Let us first consider the question of the dependence of the turbulence coefficient on wind speed.

During 1954–1955, aboard the expedition vessel *Yulii Shokalskii*, many-hour hydrological stations were carried out in order to obtain initial data for calculating the turbulence coefficient. The observations were made at a distance of 10–12 miles from the shore, at a depth of about 200 m, with the vessel anchored. During the entire period of investigation, 16 daily stations were completed, with measurements at different horizons of temperature, salinity, and current velocity. At the same time, actinometric and gradient meteorological observations were carried out.

The data from these observations were subsequently used to compute the vertical turbulence coefficient K.

The computation of K was carried out by the formula following from the basic equation of vertical heat propagation in the sea:

$$K_{z_1} = \frac{\int_{z_1}^{H} \frac{\partial\theta}{\partial t}\,dz - \frac{1-A}{c\rho} I_0 \beta \int_{z_1}^{H} e^{-\beta z}\,dz}{(\partial\theta/\partial z)_{z_1}}; \qquad (1)$$

where K_{z_1} is the turbulence coefficient, with dimension $\mathrm{cm}^2 \cdot \mathrm{sec}^{-1}$; θ is sea temperature; t is time; H is the depth of propagation of diurnal temperature oscillations; A is the albedo of the water surface; β is the coefficient of absorption of radiation; I_0 is the total radiation incident on the sea surface; and c and ρ are, respectively, the heat capacity and density of seawater.

In computing K by formula (1), observations corresponding to an established wind were used (a wind of a given speed had been blowing for no less than 4 hours before the beginning of the observations).

Figure 1 gives the dependence of K on wind speed V, constructed from the materials of the observations mentioned above. The curve obtained by us agrees in general outline with the analogous curve obtained by Stommel for the Gulf of Mexico ([1]), and with Sverdrup' s data ([2]).

2. Let us turn to consideration of the question of the dependence of K on the elements of sea waves. Theoretically this problem was first considered by S. V. Dobroklonsky ([3]). Dobroklonsky proceeded from Prandtl' s concept of the mixing length in turbulent exchange. In the semi-empirical theory

the turbulence, the dependence of the mixing length on the derivatives of the averaged velocity is introduced:

$$l = k\frac{\partial u/\partial z}{\partial^2 u/\partial z^2}, \tag{2}$$

where $k = 0.4$ is the von Kármán constant; u is the horizontal component of the averaged velocity. In accordance with the semi-empirical theory of turbulence, the coefficient of turbulence is determined by the mixing-path length and the gradient of the averaged velocity:

$$K = l^2\frac{\partial u}{\partial z}. \tag{3}$$

To determine the gradients of the averaged velocity, Dobroklonskii used the trochoidal theory of waves. The approximate expression he obtained for the coefficient of turbulence near the sea surface may be represented in the form

$$K \approx p\frac{h^2}{T}, \tag{4}$$

where p is a dimensionless coefficient equal to $2.8\cdot10^{-2}$; h and T are, respectively, the height and the period of the wave.

In Bowden' s work ([4]), the question of the influence of waves on turbulent exchange in the sea was considered. Proceeding from the fact that wave motion is characterized by three parameters—the wave amplitude a, the wavelength λ, and the period T—one obtains

$$K = p' a^\alpha \lambda^\beta T^\gamma,$$

where p' is a dimensionless coefficient.

From the dimension of K, as a simple solution of this problem Bowden assumes

$$\alpha = \beta = 1, \quad \gamma = -1;$$

then

$$K = p' \frac{a\lambda}{T},$$

or

$$K = p'ac, \tag{5}$$

where c is the velocity of wave propagation. As the author notes, expression (5) is meaningful in the case where p' is a constant quantity.

Fig. 1. Dependence of the coefficient of turbulence on wind speed

Thus, to the question of the dependence of K on the parameters of sea waves there are two answers, both of which satisfy the dimension of K.

To check the correctness of expressions (4) and (5), the curve of the dependence of K on wind speed (Fig. 1), obtained by us on the basis of measurements at sea, was used. In fact, checking these expressions comes down to establishing in which of the proposed expressions the coefficient p is constant.

Since we had no instrumental measurements of wave elements, for their calculation we used known dependences, which

allow the elements of waves to be computed from the wind speed and the wave fetch. The further course of the calculation of the coefficients p and p'

$$p = \frac{K}{h^2 T}; \qquad p' = \frac{K}{ca}$$

is given in Table 1, where v is the wind speed; the values of K are taken from the curve in Fig. 1; h is the wave height; T is the wave period; c is the wave-propagation speed; a is the wave amplitude.

Table 1

v, cm sec^{-1}	K, cm sec^{-1}	h, cm	T, sec	c, cm sec^{-1}	h^2/T, cm^2 sec^{-1}	ca, cm^2 sec^{-1}	$p \cdot 10^2$	$p' \cdot 10^2$
1.0	3.3	10	0.45	70	222	350	1.49	0.94
1.5	4.7	15	0.67	105	336	790	1.40	0.60
2.0	7.5	20	0.90	140	445	1400	1.68	0.54
2.5	10.2	26	1.12	175	603	2270	1.69	0.45
3.0	13.0	32	1.35	210	760	3360	1.74	0.39
3.5	16.8	43	1.57	245	1180	5270	1.42	0.32
4.0	21.2	50	1.80	280	1390	7000	1.52	0.30
4.5	26.2	60	2.01	314	1790	9400	1.46	0.28

v, cm sec^{-1}	K, cm sec^{-1}	h, cm	T, sec	c, cm sec^{-1}	h^2/T, cm^2 sec^{-1}	ca, cm^2 sec^{-1}	$p \cdot 10^2$	$p' \cdot 10^2$
5.0	33.5	70	2.24	350	2200	12250	1.45	0.27

As follows from Table 1, the coefficient p' decreases as the degree of wave development increases. The coefficient p is constant (within the limits of measurement accuracy); the mean value of p is $1.5 \cdot 10^{-2}$. This result should be regarded as confirmation by direct measurements at sea of an important regularity theoretically established by S. V. Dobroklonskii.

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Note: Figure translations are in progress. See original paper for figures.