ON THE PROPAGATION OF A STRONG SPHERICAL EXPLOSION WAVE IN A HEAT-CONDUCTING GAS*

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Abstract

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ON THE PROPAGATION OF A STRONG SPHERICAL EXPLOSION WAVE IN A HEAT-CONDUCTING GAS*

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The present note is devoted to the self-similar problem of a strong point explosion in a perfect gas with allowance for heat conduction. In accordance with the formulation of the problem of a point explosion ([1]), we assume that at the instant of time $t = 0$, in a gas at rest, a large but finite energy E_0 was instantaneously released at a point, i.e. an explosion occurred. As a result of this, a spherical shock wave propagates through the gas. The density of the gas at rest is ρ_1. We neglect the initial gas pressure p_1.

The self-similar problem of a strong explosion with allowance for heat conduction in cylindrical symmetry was investigated by I. O. Bezhaev ([2]). It is also known ([1,3]) that, in order to preserve the self-similarity of the problem in the spherical case, one must adopt the following dependence of the coefficient of thermal conductivity on temperature: $\varkappa = \varkappa_1 T^{1/2}$ ($\varkappa_1$ is a certain constant).

1. We have taken the initial system of equations of gas dynamics for the problem under consideration in the form

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) + R\frac{\partial(\rho T)}{\partial r} = 0, \qquad \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + \frac{2\rho u}{r} = 0,$$

$$-\varkappa_1 T^{1/2}\frac{\partial T}{\partial r} - R\rho T u + \left(\frac{2}{5}\frac{r}{t} - u\right)\left(\frac{\rho u^2}{2} + R\frac{\rho T}{\gamma - 1}\right) = 0, \tag{1}$$

where u is the velocity; R is the gas constant; r is the distance from the center of symmetry; γ is the Poisson adiabatic exponent. In system (1), the first equation is the momentum equation, the second equation is the continuity equation, and the third equation is the energy integral, found by G. M. Bam-Zelikovich ([3]) and taken in place of the heat-inflow equation.

The boundary conditions of the problem are: the condition at the center of the explosion

$$u(0, t) = 0 \tag{2}$$

and the conditions at the shock wave:

$$\rho_2(c-u_2)=\rho_1 c, \qquad u_2(c-u_2)-RT_2=0,$$

$$\rho_2(c-u_2)\left(\frac{u_2^2}{2}+R\frac{\rho_2 T_2}{\gamma-1}\right)-R\rho_2 T_2 u_2+\varkappa_2\left(\frac{\partial T}{\partial r}\right)_2=0, \tag{3}$$

where c is the shock-wave velocity, and the subscript 2 refers to quantities immediately behind the shock front.

In (1) and (3), the pressure has been eliminated by means of the equation of state $p=R\rho T$, and it has been taken into account that $u_1=0$, $p_1=0$, $T_1=0$. By virtue of the law

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for the energy conservation of the sought functions the relation must also be satisfied

$$E_0=4\pi\int_0^{r_2}\left(\frac{\rho u^2}{2}+\frac{R\rho T}{\gamma-1}\right)r^2\,dr. \tag{4}$$

For the radius and velocity of the shock wave we have the dependences (1)

$$r_2=\lambda^*\left(\frac{\alpha E_0}{\rho_1}\right)^{1/5}t^{2/5}, \qquad c=\frac{2}{5}\frac{r_2}{t}, \tag{5}$$

where λ^* is the value of r/r_2 at the discontinuity; α is a dimensionless constant that will be determined below.

2. Introduce the dimensionless variables f, g, θ, λ by the formulas

$$u=\frac{2c}{\gamma+1}f(\lambda),$$

$$\rho=\frac{\gamma+1}{\gamma-1}\rho_1 g(\lambda),$$

$$T=\frac{2c^2}{R}\frac{\gamma-1}{(\gamma+1)^2}\theta(\lambda),$$

$$\lambda=r/r_2.$$

Fig. 1

Figure 1: Fig. 1

Fig. 1

By virtue of the self-similarity of the problem, system (1) is equivalent to the following system of ordinary differential equations for the dimensionless functions f, g, θ:

$$\left(f - \frac{\gamma+1}{2}\lambda\right) f' - \frac{3(\gamma+1)}{4} f + \frac{\gamma-1}{2g}(g\theta)' = 0,$$

$$\left(f - \frac{\gamma+1}{2}\lambda\right) g' + g\left(f' + \frac{2f}{\lambda}\right) = 0, \tag{6}$$

$$A\theta^{1/2}\theta' - \frac{12(\gamma-1)}{\gamma+1} g\theta f + 6g\left(\lambda - \frac{2}{\gamma+1} f\right)(f^2+\theta) = 0,$$

where A is a dimensionless constant:

$$A = \frac{6\cdot 2^{1/2}(\gamma-1)^{11/10}(\gamma+1)^{-11/5}x_1}{R^{7/10}(0.4\rho_1)^{2/5}(\alpha E_0)^{1/5}(\lambda^*)^{5/2}}.$$

The boundary conditions (2), (3) and relation (4) can also be transformed to dimensionless variables:

$$g_2\left(1 - \frac{2f_2}{\gamma+1}\right) = \frac{\gamma-1}{\gamma+1}, \qquad f_2\left(1 - \frac{2f_2}{\gamma+1}\right) = \frac{\gamma-1}{\gamma+1}\theta_2,$$

$$A\theta_2^{1/2}\theta_2' - \frac{12(\gamma-1)}{\gamma+1}\theta_2 g_2 f_2 + 6g_2\left(1 - \frac{2f_2}{\gamma+1}\right)(f_2^2+\theta_2) = 0, \tag{7}$$

$$f(0) = 0,$$

$$\frac{1}{\alpha} = \frac{8\pi(0.4)^2(\lambda^*)^5}{(\gamma+1)(\gamma-1)}\int_0^{\lambda^*} g(f^2+\theta)\lambda^2\, d\lambda. \tag{8}$$

If one takes $\lambda^* = 1$, then after the functions $f(\lambda), g(\lambda), \theta(\lambda)$ have been found, the value of α is determined from formula (8). Thus, in dimensionless variables the problem reduces to integrating system (6) with boundary conditions (7) at $\lambda = 1$ and $\lambda = 0$.

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

3. It follows from the preceding that the system of equations (6) and the boundary conditions (7) depend parametrically on the dimensionless constants γ and A.

Fig. 2

As an example, we carried out a numerical solution of the problem for $\gamma = 1.4$ and $A = 1.048$. The indicated value of the coefficient A corresponds, under normal conditions of an explosion in air, to a thermal-conductivity coefficient many times greater than the coefficient of the ordinary molecular thermal conductivity of air. Thus, we assume that heat exchange between gas particles is more intense than the heat exchange due to molecular thermal conductivity.

Since at the center of symmetry the system (6) has a singularity, the following asymptotic formulas were used for computing the characteristics near the center:

$$f = 1.44\,\delta_0^{-1/6}\,\frac{\alpha_0}{A}\lambda^3, \qquad g = \alpha_0 + 3\delta_0^{-1/6}\,\frac{\alpha_0^2}{A}\lambda^2, \qquad \theta = \delta_0 - 3\delta_0^{5/6}\,\frac{\alpha_0}{A}\lambda^2,$$

where $\alpha_0 = g(0)$, $\delta_0 = \theta(0)$.

At the shock-wave front and at the center of the explosion the following numerical values were obtained:

$$f(1) = 0.730, \qquad g(1) = 0.425, \qquad \theta(1) = 1.716, \qquad f(0) = 0, \qquad g(0) = 0.052,$$

$$\theta(0) = 2.040.$$

For the quantity α the value $\alpha = 1.999$ was found. The results of solving the problem are presented in the form of graphs of the dependence of the functions u/u_2, T/T_2, ρ/ρ_2, p/p_2 on λ (see Figs. 1, 2, and 3, curves a). For comparison, the corresponding graphs obtained in solving the problem under the assumption of adiabatic flow ([1]) (curves b) and "isothermal" flow ([5]) (curves v) are also given here.

Fig. 3

4. The solution obtained represents a new model for describing the initial stage of development of a strong explosion. The gas set in motion by the shock wave moves in the direction away from the center and is contained within a sphere whose radius increases with time according to law (5).

The temperature at the center of the explosion is finite and decreases monotonically with increasing r, from a maximum at the center to a minimum value at the shock wave. The temperature gradient at the center of the explosion is zero and has its greatest, in mo–

zero value at the shock wave. For the values $E_0 = 8.5 \cdot 10^{20}$ erg, $\rho_1 = 0.00125$ g/cm^3, and the time $t = 0.001$ sec, we have $T(r_2, t) = 2.38 \cdot 10^5\,^\circ$K, $T(0, t) = 2.83 \cdot 10^5\,^\circ$K. These results give an idea of the temperatures in the flow region for large explosion energies.

The particle velocities near the center are small. For $r < 0.5r_2$, the gas particles are close to a state of rest. The pressure distribution remains qualitatively the same as in the case of adiabatic and "isothermal" model flow.

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Note: Figure translations are in progress. See original paper for figures.