PHYSICS

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1957

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Abstract

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PHYSICS

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ON THE THEORY OF STANDING STRIATIONS IN THE POSITIVE COLUMN OF A GAS DISCHARGE

(Presented by Academician N. N. Bogolyubov, 14 IX 1956)

Many works have been devoted to the study of standing and traveling striations in the positive column of a gas discharge. As was shown ([1]), the theoretical study of striations in a gas discharge must be based on the diffusion equations for the charged particles constituting the plasma.

However, it has become clear that the theory, as a rule, leads to an explanation only of traveling periodicities, but does not explain standing striations. Watanabe and Oleson ([2]), on the basis of a system of equations consisting of diffusion equations for charged particles and Poisson' s equation for the electric field, obtain expressions for the wave number and the velocity of a wave propagating in the plasma and, analyzing them, point out that under no conditions is the wave velocity equal to zero, unless the wave period is extraordinarily large. In other words, their paper gives a theory of traveling layers.

In Driverstein' s work ([3]), the same problem is solved for a quasineutral plasma, but with allowance for the process of thermal diffusion and for the dependence of the ionization coefficient on the electron temperature. His initial system of equations includes the diffusion equation for charged particles, the energy-conservation equation for electrons, and, valid for a quasineutral plasma, the equality of the gradients of the electron and ion currents. The theory, as the author notes, is unsatisfactory from the standpoint of comparison with experiment, poorly reflects the physical picture of the phenomenon, and does not explain the existence of standing striations. Among the simplifying assumptions, Driverstein uses the hypothesis that the regime of Schottky radial ambipolar diffusion, characteristic of a homogeneous positive column, is preserved also in the case of a stratified column. In this case all particles produced as a result of ionization remain in the same transverse cross section of the tube and can be carried away only by the radial flux of ambipolar diffusion. Driverstein, however, points out that additional allowance for ambipolar diffusion along the axis of the tube will lead to different results and, possibly, will make it possible to explain the dependence of the period of standing striations on the radius. However, this was not done either by Driverstein himself or by his followers.

In a recent work Chapnik ([4]) solved the same problem, but by the method of perturbation theory with allowance for deviations from quasineutrality. He retained Driverstein' s rather arbitrary hypothesis that the particles arising during ionization are carried away only by the radial flux of ambipolar diffusion, and thus eliminated them from participation in the formation of the longitudinal stratification. The remaining terms gave, as Chapnik showed, a periodic solution for gases in which the electron collision cross section is inversely proportional to their velocity. In this case, in contrast to Driverstein, Chapnik obtains a dispersion equation for a standing periodicity, due primarily to the process of thermal diffusion.

It should be noted that standing striations in a discharge are also observed in gases in which the electron collision cross section is directly proportio-

is proportional to their velocity, and also in gases with a thermodiffusion coefficient equal to zero. In argon, for example, standing strata both in the region of decreasing and in the region of increasing dependence of the cross section on velocity do not differ from one another either in appearance or in the laws to which they obey. Thus, although thermodiffusion undoubtedly plays a role in a gas discharge (for those gases for which the thermodiffusion coefficient is not zero), Chappnik' s explanation of standing strata cannot be regarded as acceptable. Nor is the numerical agreement with experiment of the striation period, based on the fulfillment of the Schottky relation, in the two cases indicated by Chappnik a sufficient proof.

Up to now, a number of well-known regularities obeyed by the period of standing strata have not received a theoretical explanation; for example, the dependence of the striation period on the radius at constant discharge current, or the dependence of the striation period on the temperature of the electron gas at constant gas pressure.

Meanwhile, on the basis of the indicated diffusion equations, quite rigorous expressions can be obtained for the period of standing strata, agreeing with experiment in order of magnitude and making it possible to explain the dependence of the striation period on the tube radius, on the magnitude of the discharge current, etc.

Let us write down the initial system of equations for a striated discharge:

$$\frac{\partial n_1}{\partial t} - \frac{\partial (n_1 u_{1\alpha})}{\partial x_\alpha} - D_1 \frac{\partial^2 n_1}{\partial x_\alpha^2} = \beta n_1 - R, \tag{1}$$

$$\frac{\partial n_2}{\partial t} + \frac{\partial (n_2 u_{2\alpha})}{\partial x_\alpha} - D_2 \frac{\partial^2 n_2}{\partial x_\alpha^2} = \beta n_1 - R, \tag{2}$$

$$\frac{\partial E_\alpha}{\partial x_\alpha} = 4\pi e (n_2 - n_1). \tag{3}$$

Here index 1 refers to electrons, index 2 to ions; $u_{1\alpha}$ and $u_{2\alpha}$ are the drift velocities of electrons and ions, respectively; D_1 and D_2 are the diffusion coefficients; β is the ionization coefficient; R is volume recombination in the discharge; n_1 and n_2 are the concentrations, respectively, of electrons and ions.

Let us align the axis of the tube with the x-axis. In the central part of the tube we may regard the dependence of the particle concentration on coordinates other than x as weak: $x_\alpha = x$, and for volume recombination adopt the relation $R = \beta n_0$, where n_0 is the equilibrium concentration of charged particles in the central part of an unstriated discharge, which is taken as the zero approximation.

Putting

$$\begin{aligned} n_1 &= n_0 + \nu_1 e^{i(\omega t + kx)} \quad (\nu_1 \ll n_0); \\ n_2 &= n_0 + \nu_2 e^{i(\omega t + kx)} \quad [(\nu_2 \ll n_0) \end{aligned} \tag{4}$$

and taking into account that $u_1 = \mu_1 E$, $u_2 = \mu_2 E$, where μ_1 and μ_2 are the mobilities of electrons and ions, we obtain:

$$\nu_1 \left(i\omega - ik + D_1 k^2 - \beta + \frac{\omega_e^2}{a_1} \right) - \nu_2 \frac{\omega_e^2}{a_1} = 0, \tag{5}$$

$$\nu_2 \left(i\omega + u_2 ik + D_2 k^2 + \frac{\omega_i^2}{a_2} \right) - \left(\beta + \frac{\omega_i^2}{a_2} \right) \nu_1 = 0, \tag{6}$$

where

$$\frac{\omega_e^2}{a_1} = 4\pi e n_0 \mu_1 = \frac{D_1}{d_1^2}; \qquad \frac{\omega_i^2}{a_2} = 4\pi e n_0 \mu_2 = \frac{D_2}{d_2^2},$$

d_1 and d_2 are the Debye radii for electrons and ions.

From the condition for the existence of a nontrivial solution of the system we obtain the following dispersion equation:

$$\begin{aligned} (i\omega - u_1 ik + D_1 k^2 + 4\pi e n_0 \mu_1 - \beta)(i\omega + u_2 ik + D_2 k^2 + 4\pi e n_0 \mu_2) - \\ -4\pi e n_0 \mu_1 (\beta + 4\pi e n_0 \mu_2) = 0. \end{aligned} \tag{7}$$

If one takes into account that in an ordinary gas-discharge plasma the Debye radius is very small, of the order of $10^{-2} \div 10^{-3}$ cm (which corresponds to a sufficiently high degree of plasma ionization: 10^{-5}), then the terms $4\pi e n_0 \mu_1$ and $4\pi e n_0 \mu_2$ in the equation are much larger than the other terms. We then obtain:

$$\left(D_1\frac{\mu_2}{\mu_1}+D_2\right)k^2+(i\omega-\beta)\left(1+\frac{\mu_2}{\mu_1}\right)=0. \tag{8}$$

$$|\gamma_1-\gamma_2|\ll|\gamma_1|.$$

Under the condition

$$k^2=\frac{\beta\left(1+\frac{\mu_2}{\mu_1}\right)}{D_1\frac{\mu_2}{\mu_1}+D_2} \tag{9}$$

we have a stable quasineutral standing periodicity*:

$$l=\frac{2\pi}{\sqrt{\frac{\beta}{D_1\frac{\mu_2}{\mu_1}+D_2}\left(1+\frac{\mu_2}{\mu_1}\right)}}. \tag{10}$$

On the basis of a more complete system of equations, including the equation of conservation of particle energy and taking into account thermodiffusion, the dependence of the equation parameters on temperature, etc., one can obtain a more complicated dispersion equation which, however, assumes the simpler form indicated above (8) when those real conditions under which striations are observed are taken into account.

For a known gas pressure and electron temperature, with a specified tube radius and for a specified gas, it is easy to calculate the period of standing striations from formula (10). The results of such a calculation for two gases—argon and neon—at a constant discharge current are given in Table 1. In

Table 1

	Argon, $p=10$ **mm Hg**	**Argon,** $p=10$ **mm Hg**	**Argon,** $p=10$ **mm Hg**	**Argon,** $p=10$ **mm Hg**	**Argon,** $p=10$ **mm Hg**	**Argon,** $p=10$ **mm Hg**	**Neon,** $p=10$ **mm Hg**	**Neon,** $p=10$ **mm Hg**	**Neon,** $p=10$ **mm Hg**	**Neon,** $p=10$ **mm Hg**	**Neon,** $p=10$ **mm Hg**	**Neon,** $p=10$ **mm Hg**
r_0, cm	0.1	0.2	0.4	0.6	0.8	1.0	0.1	0.2	0.4	0.6	0.8	1.0
l_t, cm	0.35	1.5	3.6	4.8	5.7	–	0.5	1.14	2.2	3.0	3.9	4.82
l_e, cm	0.77	1.4	2.46	3.46	4.4	–	0.85	1.53	2.7	3.8	4.8	5.8

the calculation we used the known experimental dependence of the electron temperature on the pressure and the tube radius ([9]). Owing to the absence of data concerning changes in the temperature of the neutral gas and ions, in the calculations we considered the ion mobility μ_2 to be constant. Taking into account the change in ion mobility with the change in gas temperature leads to a decrease in the steepness of the dependence $l_t = f(r_0)$, especially for argon, whose temperature changes with change of tube radius are especially

* The theory of Watanabe and Oleson ([2]) suffers precisely from the failure to take into account the force of interaction of electrons and ions in real plasmas, which is especially clearly seen in their article from the expression for the velocity of the wave. If one takes into account that the terms $4\pi e n_0 \mu_2$ and $4\pi e n_0 \mu_1$ are much larger than all the other terms of the expression, then the denominator of the fraction is large, and the wave velocity is equal to zero regardless of the magnitude of the striation period. Thus, the conclusions of Watanabe and Oleson apply to weakly charged plasmas, in which the Debye radius is of the order of the striation period and much greater than the mean free path.

large. In the same table, for comparison, values of l_e are given for the dependence of the striation period on the tube radius in a high-frequency discharge at constant discharge-current density, according to the work of A. A. Zaitsev and Kh. A. Dzherpetov ([5]).

It should be noted that in this simplest case the dispersion equation for a high-frequency discharge completely coincides with dispersion equation (8). It is only necessary that the amplitude of the high-frequency field E_0 not be too large, obeying the condition

$$\frac{\mu_1 E_0}{c_1} < \frac{\lambda_1}{l}, \tag{11}$$

where λ_1 is the mean free path for electrons, and c_1 is their thermal velocity. Comparison shows the same character of the dependence and the closeness of the experimental and calculated striation periods.

Table 2

Neon, $p = 1.8$ mm Hg, $r_0 = 3.1$ cm

	T_1, °K	T_1, °K	T_1, °K	T_1, °K
	$3.5 \cdot 10^4$	$3.3 \cdot 10^4$	$2.85 \cdot 10^4$	$2 \cdot 10^4$
l_T, cm	1.3	1.8	2.3	4.5
l_e, cm	3.6	4.0	4.5	–

Table 2 gives the dependence of the striation period for neon on the temperature of the electron gas (which changed as the discharge current was varied) at constant pressure. For comparison, the same table presents experimental data for neon obtained by Zaitsev and Dzherpetov for approximately the same range of variation of the electron temperature and having the same character of dependence.

We established the correspondence between the discharge-current density and the electron temperature with the aid of graphs published in papers ([7,8]). Because of the small scale of the graphs and the different experimental conditions, this correspondence may be regarded as approximate.

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Received
12 IX 1956

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Note: Figure translations are in progress. See original paper for figures.