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## Abstract

## Full Text

*Physics*

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# QUADRUPOLE MOMENTS OF NUCLEI

*(Presented by Academician N. N. Bogolyubov on 20 VI 1957)*

The electric quadrupole moments of nuclei have been studied in works <sup>(1-8)</sup>. Since within the framework of the shell model this question has not received a definitive solution, the author decided to consider it once again. We shall represent the wave function of the nucleus in the form of a linear combination of Slater functions composed of one-nucleon wave functions. In the case of strong spin-orbit coupling, the latter have the form <sup>(9)</sup>:

$$\psi_{\alpha_i}(\bar{r}_i, s_i, t_i) = R_{\beta_i}(r_i) \begin{vmatrix} a_i Y_{l_i, m_i}(\theta_i, \varphi_i) \\ b_i Y_{l_i, m_i+1}(\theta_i, \varphi_i) \end{vmatrix} T_{\tau_i}(t_i), \quad (1)$$

where  $\bar{r}_i, s_i, t_i$  are, respectively, the spatial, spin, and isotopic-spin coordinates of the  $i$ -th nucleon;  $\alpha_i = (n_i, j_i, j_{zi}, \tau_{zi})$ ,  $\beta_i = (n_i, l_i, j_i)$  are sets of quantum numbers characterizing the state of the nucleon;  $n_i$  is the principal quantum number;  $l_i$  is the orbital quantum number;  $j_{zi}$  is the quantum number of the projection of the total angular momentum on an arbitrary  $z$ -axis;  $m_i$  is the magnetic quantum number;  $\tau_{zi}$  is the quantum number of the isotopic spin of the  $i$ -th nucleon;  $R_{\beta_i}(r_i)$  is the radial wave function;  $Y_{l,m}(\theta, \varphi)$  is a spherical function;  $T_{\tau_i}(t_i)$  is the wave function of isotopic spin;  $a_i$  and  $b_i$  are known coefficients <sup>(9)</sup>.

The coefficients in the expansion of the nuclear wave function in Slater functions are found from the conservation laws for the projection of angular momentum on the  $z$ -axis, the square of angular momentum, and the square of the isotopic spin of the nucleus. Using such a wave function and the general formula for the quadrupole moment <sup>(10)</sup>, we find the formula for the quadrupole moment of the nucleus:

$$Q_{zz} = \sum_{i,k} |c_k|^2 \frac{A_i^{(k)}}{2j_i^{(k)}(j_i^{(k)} + 1)} [j_i^{(k)}(j_i^{(k)} + 1) - 3j_{zi}^{(k)2}] + \sum_{\substack{i,k,j \\ (k \neq j)}} c_k^* c_j \delta_{\tau_j^i, \tau_k^i} \prod_{\substack{n \\ (n \neq i)}} \delta_{\alpha_n^k, \alpha_n^j} A_i^{(k,j)} B_i^{(k,j)}, \quad (2)$$

where  $i$  is the number of the proton;  $k$  and  $j$  are the numbers of the Slater functions;  $c_k$  is the amplitude of the  $k$ -th Slater function; the number  $n$  runs over all integers from 1 to  $A$  (except  $n = i$ );  $A$  is the number of particles in the nucleus;

$$A_i^{(k)} = \int R_{\beta_i^{(k)}}^2(r) r^4 dr; \quad A_i^{(k,j)} = \int R_{\beta_i^{(k)}}(r) R_{\beta_i^{(j)}}(r) r^4 dr; \quad (3)$$

$$B_i^{(k,j)} = \int (3 \cos^2 \theta - 1) \left\{ a_i^{(k)} a_i^{(j)*} Y_{l_i^{(k)}, m_i^{(k)}}^* Y_{l_i^{(j)}, m_i^{(j)}} + \right. \\ \left. + b_i^{(k)} b_i^{(j)*} Y_{l_i^{(k)}, m_i^{(k)}+1}^* Y_{l_i^{(j)}, m_i^{(j)}+1} \right\} d\Omega_i. \quad (4)$$

It can be shown that the value of the coefficient  $A_i^{(k)}$  lies in the interval  $(R^2/3, R^2)$ , and its mean value is approximately equal to  $0.6 R^2$ , where  $R$  is the nuclear radius.

Table 1 gives the electric quadrupole moments of certain light nuclei calculated by formula (2), under the assumption that  $A_i^{(k)} \approx 0.6 R^2$  (the calculation was carried out without taking configuration admixtures into account).

**Table 1**

Nucleus	$3\text{Li}^7$	$4\text{Be}^7$	$5\text{B}^9$	$4\text{Be}^9$	$5\text{B}^{11}$	$3\text{Al}^{27}$
$Q_{zz}$ theor.	-0.013	-0.012	0.016	0.025	0.0250	0.07
$Q_{zz}$ exper.	< 0	$Q_{zz}(\text{Li}^7) \sim \sim$ $Q_{zz}(\text{Be}^7)$	-	$\simeq (0.02)$	$0.0355 \pm 2$	$0.149 \pm 2$
Nucleus	$29\text{Cu}^{63}$	$29\text{Cu}^{65}$	$17\text{Cl}^{37}$	$31\text{Ga}^{69}$	$31\text{Ga}^{71}$	$83\text{Bi}^{209}$
$Q_{zz}$ theor.	-0.085	-0.086	-0.059	0.09	0.092	-0.35
$Q_{zz}$ exper.	$-0.157 \pm 31$	$-0.145 \pm 29$	$-0.06635 \pm 2$	0.243	0.152	-0.4

The last (triple) sum in formula (2) shows that, in the presence of mixed shell configurations, an additional electric quadrupole moment arises. Thus, for example, analysis of shell structures indicates a close location of the proton levels  $4d_{5/2}$ ,  $6g_{7/2}$ , and  $6h_{11/2}$ . Such levels begin to be filled by protons almost simultaneously. Therefore the protons filling these levels enter into complex states that are linear combinations of the states  $4d_{5/2}$ ,  $5g_{7/2}$ , and  $6h_{11/2}$ . It may be

assumed that such complex shell configurations are responsible for at least part of the anomalously large positive electric quadrupole moment of certain nuclei with  $Z > 50$ .

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*Note: Figure translations are in progress. See original paper for figures.*

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