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Abstract

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PHYSICS

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CALCULATION OF THE MAGNETIZATION OF A UNIAXIAL FERRITE AT LOW TEMPERATURES

(Presented by Academician N. N. Bogolyubov, 5 IV 1957)

The energy of a uniaxial ferrite consisting of two ferromagnetic sublattices ([1]) can be represented in the following form ([2]):

$$\widetilde{H} = -\frac{1}{2}\sum_{h_1 h_2} I(h_1 h_2)(\mathbf{S}_{h_1}\mathbf{S}_{h_2}) - \frac{1}{2}\sum_{h_1 h_2} \Delta(h_1 h_2) S^z_{h_1} S^z_{h_2}, \tag{1}$$

where $\Delta(h_1 h_2) = I_{zz}(h_1 h_2) - I_{xx}(h_1 h_2)$, $I_{xx}(h_1 h_2) = I_{yy}(h_1 h_2)$; S_h is the spin of site h, and OZ is the axis of easy magnetization. The index h runs through the values f_i $(i = 1, \dots, N_1)$ for one ferromagnetic sublattice and g_i $(i = 1, \dots, N_2)$ for the other.

Let the resultant spin of the first sublattice be S_1, and that of the second S_2. We shall further assume that the difference $S_1 - S_2$ is of the same order as S_1 and S_2. In the presence of two sublattices, each ion of one sublattice has as its nearest neighbor a magnetic ion of the other sublattice. It is natural to expect that the exchange interaction between ions of different sublattices is considerably greater than the magnetic interaction, i.e.,

$$\left|\frac{\Delta(h_1 h_2)}{I(h_1 h_2)}\right| \ll 1. \tag{2}$$

We shall take the ratio (2) of the magnetic interaction to the exchange interaction as a small parameter. At low temperatures, using the method of approximate second quantization of Bogolyubov–Tyablikov ([2,3]), the Hamiltonian can be transformed to the form

$$\widetilde{H} = E_0 + \Delta E_0 + \widetilde{H}_2. \tag{3}$$

The meaning of E_0, ΔE_0, and $\widetilde{H}_2$ will become clear if the spin operators, in accordance with the above-mentioned method, are represented in the form ([3])

$$S_h^\alpha = \sigma_h^\alpha \left(1 - \frac{2n_h}{\sigma_h}\right) + A_h^\alpha b_h + A_h^{\alpha *} b_h^+ \qquad (\alpha = x, y, z); \tag{4}$$

n_h is the deviation of the spin from the position it occupied at absolute zero; b_h, b_h^+ are Bose-statistics operators, $b_h^+ b_h = n_h$. The vector $\vec{\sigma}_h$ is determined from the requirement of the minimum of E_0, under the condition that $|\vec{S}_h| = |\vec{\sigma}_h|$. $A_h^\alpha, A_h^{\alpha *}$ are certain coefficients which include, in particular, the field, if such a field is present.

The first two terms of the Hamiltonian, E_0 and ΔE_0, do not contain the operators b_h, b_h^+; the term $\widetilde{H}_2$ is quadratic with respect to b_h, b_h^+. In the first approximation in the anisotropy parameter (2), at absolute zero the energy of the system $E_0 + \Delta E_0$ corresponds to a minimum when the spins of one sublattice are directed exactly opposite to the spins of the other sublattice.

In a magnetic field the spins rotate, remaining antiparallel. The angle with the axis of easy magnetization φ is determined from the equation

$$\mu H_x - \frac{\sin\varphi}{\varkappa} = \operatorname{tg}\varphi \cdot \mu H_z, \tag{5}$$

where $I_{11} = \sum_{f_1} I(f_1 f_2)$; $I_{12} = \sum_g I(f, g)$; $\Delta_{12} = \sum_g \Delta(f, g)$, etc.;

$$\varkappa = \frac{(S_1 - S_2) N_1 N_2}{\Delta_{11} S_1^2 N_2^2 + \Delta_{22} S_2^2 N_1^2 - 2 S_1 S_2 \Delta_{12} N_1}. \tag{6}$$

This is valid for fields of the order of the anisotropy fields $\mu H \sim |\Delta_{12}|$. For fields of the order of the exchange fields, however, the mutual antiparallel orientation, as shown in (3), is destroyed. In the case of a field directed perpendicular to the anisotropy axis (or parallel to it), the direction of the magnetization vector can be determined exactly

$$\sin\varphi_1 = -\frac{n_1 \xi_2 - n_2(1-\omega)}{\omega(2-\omega)}, \qquad \sin\varphi_2 = -\frac{n_2 \xi_1 - n_1(1-\omega)}{\omega(2-\omega)} \quad (\Delta_{11} = \Delta_{22} = 0), \tag{7}$$

where φ_1 and φ_2 are the angles between the anisotropy axis and the magnetization vectors of the first and second sublattices. ξ_1, ξ_2, ω, n_1, n_2 are defined as follows:

$$\xi_1^2 = \frac{\omega(2-\omega)+n_1^2}{\omega(2-\omega)+n_2^2}; \qquad \xi_2^2 = \frac{\omega(2-\omega)+n_2^2}{\omega(2-\omega)+n_1^2};$$

$$\omega = \frac{\Delta_{12}}{I_{12}+\Delta_{12}}; \qquad n_1 = \frac{\mu H_x a_1}{\sigma_2(I_{12}+\Delta_{12})}; \qquad n_2 = \frac{\mu H_x a_2}{\sigma_1(I_{21}+\Delta_{21})}; \tag{8}$$

Restricting ourselves to first-order terms, we have

$$\sin\varphi_1 = \mu\varkappa H_x + \frac{\mu H_x a_1}{2\sigma_1|I_{21}|}; \qquad \sin\varphi_2 = -\mu\varkappa H_x + \frac{\mu H_x a_2}{\sigma_2|I_{12}|}; \tag{9}$$

$$a_1 = \frac{(S_1+S_2)(S_1\cos 2\varphi_0 - S_2)}{2S_2(S_1-S_2)}.$$

The magnetization of the ferrite in this case is

$$M_0 = \mu^2\varkappa H_x(S_1-S_2) + \mu^2 H_x \frac{N_2}{|I_{12}|}\frac{1}{2}(a_1+a_2). \tag{10}$$

In the case of a field parallel to the anisotropy axis, it can be shown that the spins are arranged along the axis opposite to each other up to fields of the order of the exchange fields. If, without changing the latter orientation of the field, its magnitude is varied down to zero and further, hysteresis is observed, i.e., the system of mutually opposite spins does not immediately change its position, but only upon reaching the field value $\mu H' = -1/\varkappa$. The magnetization in this case changes discontinuously from $M_0 = \mu(S_1 - S_2)$ to $M_0 = -\mu(S_1 - S_2)$.

For the magnetization in the temperature interval $|\Delta_{12}| \ll \vartheta \ll \vartheta_c \sim |\overline{I}_{12}|$, if the field is directed perpendicular to the anisotropy axis, we obtain

$$\frac{M}{V} = \frac{M_0}{V}(1+c_1T^{3/2}); \qquad \mu H \lesssim 1/\varkappa \sim |\Delta_{12}| \quad (M_0 \text{ see } (10));$$

$$\frac{M}{V} = \frac{M_0}{V}(1-c'T^{3/2}); \qquad |\Delta_{12}| \ll \mu H \ll \mu H_1 = \frac{\sqrt{\overline{I}_{12}I_{21}}}{\sqrt{N_1N_2}}; \qquad M_0 = \mu(S_1-S_2); \tag{11}$$

$$\frac{M}{V} = \frac{M_0}{V}(1+c''T^{3/2}) \qquad \mu H \lesssim \mu H_1; \qquad M_0 = \mu(S_1-S_2),$$

where the coefficients c have the form:

$$c_1 = c' \sin\varphi = \frac{\mu V \cdot 0.12}{M_0(\alpha'/k)^{3/2}}; \qquad c'' = \frac{\mu V \cdot 0.12}{M_0(\alpha''/k)}; \tag{12}$$

V is the volume; α', $\alpha'' \sim |I_{12}|$.

The magnetization in a field parallel to the anisotropy axis, with increasing temperature, decreases in small fields according to the law

$$M = M_0(1 - cT^{3/2}), \tag{12'}$$

where $c = c'$; while in large fields $\mu H \lesssim \mu H_1$ it increases:

$$M = M_0(1 + cT^{3/2}), \tag{12''}$$

where $c = c''$.

The increase of magnetization with temperature in certain fields can also be understood from a physical point of view. Thus, the increase of magnetization in small fields directed perpendicular to the anisotropy axis occurs because a deviation of the magnetic moment toward the field is energetically more favorable than against the field. Therefore, upon heating, when the moments deviate from their initial position, the "amplitude" of deviation toward the field is larger than against it.

An increase of magnetization in a ferrite can also occur because the number of spins reversed from one sublattice may be unequal to the number of spins reversed from the other. This occurs, for example, in large fields $\mu H \lesssim \mu H_1$ ([3]). Then it is more favorable to rotate the spin of the sublattice oriented opposite to the field than to change the direction of the spin of the sublattice oriented along the field.

The behavior of the magnetization qualitatively agrees with that obtained in ([4]) for uniaxial crystals of magnetite.

Above we considered the most probable case of strong exchange interaction between nearest neighbors belonging to different sublattices. However, one can consider the case in which the ferrite consists of two ferromagnetic sublattices weakly coupled to each other. In this case the magnetization is represented by a somewhat corrected sum of the magnetizations of each of the sublattices. In this case too, the magnetization behaves in qualitative agreement with experiment ([4]). However, the height of the hysteresis loop in the case of weak coupling should be greater than the corresponding loop for strong coupling, and in order of magnitude will be equal to $\mu(S_1 + S_2)$.

Apparently, this makes it possible to determine what coupling exists in a given case.

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Note: Figure translations are in progress. See original paper for figures.