ELECTRICAL ENGINEERING

1957

SovietRxiv

Abstract

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ELECTRICAL ENGINEERING

A. G. LUNTS

A METHOD FOR SYNTHESIZING A $(1, k)$-TERMINAL NETWORK

(Presented by Academician A. N. Kolmogorov, 7 VIII 1956)

As is known (^{1}), any equivalent transformation of a contact n-terminal network A, i.e., a transformation that does not change the total conductances between all possible pairs of terminals, can be effected by an algebraic transformation of the characteristic function of this multi-terminal network:

$$f_A(x_1, x_2, \dots, x_n) = \sum_{\alpha,\beta=1}^{n} a_{\alpha\beta}\bar{x}_\alpha x_\beta. \tag{1}$$

Here $a_{\alpha\beta}$ denotes the direct conductance from terminal M_α to terminal M_β. If the multi-terminal network A is regarded as a $(1, n-1)$-terminal network with input M_1 and outputs $M_2, M_3, \dots, M_n$, then an equivalent transformation of such a $(1, n-1)$-terminal network should be called a transformation that does not change the total conductances $\chi_{\alpha\beta}(A)$ $(\beta = 2, 3, \dots, n)$ between the input M_1 and any output. In the present paper it is proved that any such transformation can also be effected by an algebraic transformation of the characteristic function (1), but using the system of inequalities

$$x_1 = 1, \qquad x_\beta \leqslant \chi_{1\beta}(A) \quad (\beta = 1, 2, \dots, n). \tag{2}$$

Consider the class K of equivalent $(1, n-1)$-terminal networks, i.e., the set of those n-terminal networks A whose total conductances are equal to the given quantities $\chi_{1\beta}$:

$$\chi_{1\beta}(A) = \chi_{1\beta} \quad (\beta = 2, 3, \dots, n). \tag{3}$$

Denote

$$\sum_{\beta=1}^{n} \chi_{1\beta} x_1 \bar{x}_\beta = u, \qquad \sum_{\beta=1}^{n} \chi_{1\beta}\bar{x}_\beta + \bar{\chi}_{1\alpha} x_\beta = v,$$

where, for symmetry of notation, it is assumed that $\chi_{11} = 1$.

Theorem 1. *The characteristic functions of all multi-terminal networks in K lie in the interval $[u, v]$; this interval contains no other characteristic functions.*

Proof. Let $A \in K$, i.e., (3) holds; then

$$f_A(x_1, \dots, x_n) = \sum_{\alpha,\beta=1}^{n} \chi_{\alpha\beta}(A) x_\alpha \bar{x}_\beta \geqslant \sum_{\beta=1}^{n} \chi_{1\beta} x_1 \bar{x}_\beta = u.$$

On the other hand, from (1), from the inequality $\chi_{1\alpha}(A) \cdot \chi_{\alpha\beta}(A) \leqslant \chi_{1\beta}(A)$, we obtain $\chi_{\alpha\beta}(A) \leqslant \chi_{1\beta} + \bar{\chi}_{1\alpha}$; therefore

$$f_A(x_1, \dots, x_n) \leqslant \sum_{\alpha,\beta=1}^{n} (\chi_{1\beta} + \bar{\chi}_{1\alpha}) x_\alpha \bar{x}_\beta \leqslant \sum_{\alpha,\beta=1}^{n} \chi_{1\beta} \bar{x}_\beta + \bar{\chi}_{1\alpha} x_\alpha = v.$$

Conversely, let $u \leqslant f_A(x_1, \dots, x_n) \leqslant v$; we shall show that $A \in K$. Putting

in the inequality

$$u = \sum_{\beta=1}^{n} \chi_{1\beta} x_1 \overline{x}_\beta \leq f_A(x_1, \dots, x_n) \leq \sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{x}_\beta + \overline{\chi}_{1\beta}(A) \overline{x}_\beta$$

putting $x_\beta = \chi_{1\beta}(A)$ $(\beta = 1, 2, \dots, n)$, we obtain

$$\sum_{\beta=1}^{n} \chi_{1\beta} \overline{\chi}_{1\beta}(A) \leq 0,$$

i.e. $\chi_{1\beta} \leq \chi_{1\beta}(A)$.

Next, putting in the inequality

$$\sum_{\beta=1}^{n} \chi_{1\beta}(A) x_1 \overline{x}_\beta \leq f_A(x_1, \dots, x_n) \leq \sum_{\beta=1}^{n} \chi_{1\beta} \overline{x}_\beta + \overline{\chi}_{1\beta} x_\beta = v$$

$x_\beta = \chi_{1\beta}$ $(\beta = 1, 2, \dots, n)$, we obtain

$$\sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{\chi}_{1\beta} \leq 0,$$

i.e. $\chi_{1\beta}(A) \leq \chi_{1\beta}$. Consequently, $\chi_{1\beta}(A) = \chi_{1\beta}$, i.e. $A \in K$.

Theorem 2. In order that two $(1, n-1)$-terminal networks A and B be equivalent, i.e.

$$\chi_{1\beta}(A) = \chi_{1\beta}(B) \quad (\beta = 1, 2, \dots, n), \tag{4}$$

it is necessary and sufficient that, for all values $x_1, x_2, \dots, x_n$ satisfying the inequalities (2), their characteristic functions be equal:

$$f_A(x_1, x_2, \dots, x_n) = f_B(x_1, x_2, \dots, x_n). \tag{5}$$

Proof. Necessity. The proved inequality

$$\sum_{\beta=1}^{n} \chi_{1\beta}(A) x_1 \overline{x}_\beta \le f_A(x_1, \dots, x_n) \le \sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{x}_\beta + \overline{\chi}_{1\beta}(A) x_\beta$$

for values of the variables satisfying (2) gives

$$\sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{x}_\beta \le f_A(x_1, \dots, x_n) \le \sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{x}_\beta,$$

i.e.

$$f_A(x_1, \dots, x_n) = \sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{x}_\beta.$$

Using (4), we obtain for $f_B(x_1, \dots, x_n)$ the same expression and, consequently, equality (5).

Sufficiency. Suppose that in the inequality

$$\sum_{\beta=1}^{n} \chi_{1\beta}(B) x_1 \overline{x}_\beta \le f_B(x_1, \dots, x_n) = f_A(x_1, \dots, x_n) \le \sum_{\beta=1}^{n} \chi_{1\beta}(A) \overline{x}_\beta + \overline{\chi}_{1\beta}(A) x_\beta$$

the quantity $x_\beta = \chi_{1\beta}(A)$ $(\beta = 1, 2, \dots, n)$; then we obtain

$$\sum_{\beta=1}^{n} \chi_{1\beta}(B) \overline{\chi}_{1\beta}(A) \le 0,$$

i.e.

$$\chi_{1\beta}(B) \le \chi_{1\beta}(A). \tag{6}$$

Consequently, equality (5) holds for all values $x_1, x_2, \dots, x_n$ satisfying the inequalities $x_1 = 1$, $x_\beta \le \chi_{1\beta}(B)$ $(\beta = 1, 2, \dots, n)$; therefore, interchanging the roles of A and B, we shall also have

$$\chi_{1\beta}(A) \le \chi_{1\beta}(B). \tag{7}$$

From (6) and (7) follows the equivalence of the $(1, n-1)$-terminal networks A and B.

The theorem just proved gives a method of synthesis of a $(1, n-1)$-terminal network with given conductivities $\chi_{12}, \chi_{13}, \dots, \chi_{1n}$. First we write the expression

$$f_A(1, x_2, \dots, x_n) = \sum_{\beta=2}^{n} \chi_{1\beta}\overline{x}_\beta;$$

then, using the inequalities (2), we transform it. For example, to $f_A(1, x_2, \dots, x_n)$ one may add terms of the form $a_{\alpha\beta}x_\alpha\overline{x}_\beta$ $(\alpha, \beta = 2, 3, \dots, n)$, where the quantity $a_{\alpha\beta}$ satisfies the inequality $a_{\alpha\beta}\chi_{1\alpha} \le \chi_{1\beta}$, since in this case we shall have, for the considered values of the variables $x_2, x_3, \dots, x_n$:

$$a_{\alpha\beta}x_\alpha\overline{x}_\beta \le a_{\alpha\beta}\chi_{1\alpha}\overline{x}_\beta \le \chi_{1\beta}\overline{x}_\beta \le f_A(1, x_2, \dots, x_n).$$

It is not difficult to show that, using only this device and identical transformations, one can obtain any $(1, n-1)$-terminal network satisfying the conditions of the problem.

Example 1. Construct a $(1, 2)$-terminal network with conductivities: $\chi_{12} = a + bkd + cg + chd$, $\chi_{13} = bk + ad + aek + ch + cgd + cgek$.

We choose a_{23} and a_{32} satisfying the inequalities: $a_{23}\chi_{12} \le \chi_{13}$, $a_{32}\chi_{13} \le \chi_{12}$, i.e., $a_{23}(a + bkd + cg + chd) \le bk + ad + aek + ch + cgd + cgek$, $a_{32}(bk + ad + aek + ch + cgd + cgek) \le a + bkd + cg + chd$.

These inequalities are equivalent to the inequalities $a_{23}(a + cg) \le (d + ek) \times (a + cg) + ch + bk$, $a_{32}(bk + ch) \le d(bk + ch) + a + cg$.

One may, for example, take $a_{23} = d + ek$, $a_{32} = d$. Then, after simplification, we shall have $f_A(1, x_2, x_3) = \chi_{12}\overline{x}_2 + \chi_{13}\overline{x}_3 + a_{23}x_2\overline{x}_3 + a_{32}x_3\overline{x}_2 = (a + cg)\overline{x}_2 + (bk + ch)\overline{x}_3 + (d + ek)x_2\overline{x}_3 + dx_3\overline{x}_2$.

Further simplification can be obtained by introducing new nodes: $f_A(1, x_2, x_3) = a\overline{x}_2 + d(x_2 \circ x_3) + (ex_2 + b) \cdot k\overline{x}_3 + c \cdot (g\overline{x}_2 + h\overline{x}_3)$, where $x_2 \circ x_3$ denotes the symmetric difference, i.e., in general, $x \circ y = \overline{x}y + x\overline{y}$; $f_A(1, x_2, x_3, x_4, x_5) = a\overline{x}_2 + d(x_2 \circ x_3) + (ex_2 + b)\overline{x}_4 + kx_3x_4 + c\overline{x}_5 + (g\overline{x}_2 + h\overline{x}_3)x_5 = a\overline{x}_2 + d(x_2 \circ x_3) + ex_2\overline{x}_4 + b\overline{x}_4 + kx_3x_4 + c\overline{x}_5 + gx_5\overline{x}_2 + hx_5\overline{x}_3$, which corresponds to the circuit of Fig. 1a, where the arrows indicate the directions of the conductivities.

Fig. 1

Figure 1: Fig. 1

Fig. 1

Example 2. Construct a $(1,2)$-terminal network A without valve elements, with conductivities $\chi_{12} = c + ed + adg + ab + beg$, $\chi_{13} = b + dg + aed + ac + ceg$.

For a_{23} and a_{32}, as in the preceding example, we write the inequalities $a_{23}\chi_{12} \leq \chi_{13}$, $a_{32}\chi_{13} \leq \chi_{12}$, i.e. $a_{23}(c + ed + adg + ab + beg) \leq b + dg + aed + ac + ceg$, $a_{32}(b + dg + aed + ac + ceg) \leq c + ed + adg + ab + bed$, which are equivalent to the inequalities $a_{23}(c + de) \leq b + a(c + de) + g(d + ce)$, $a_{32}(b + dg) \leq c + a(b + dg) + e(d + bg)$.

One may, for example, take $a_{23} \leq b + a + eg$, $a_{32} \leq c + a + eg$, but since in the desired solution we must have $a_{23} = a_{32}$, then $a_{23} = a_{23}a_{32} \leq (b + a + eg) \cdot (c + a + eg) = bc + a + eg$. Taking $a_{23} = a + bc + eg$, we shall have, after simplification, $f_A(1, x_2, x_3) = \chi_{12}\overline{x}_2 + \chi_{13}\overline{x}_3 + a_{23}(x_2 \circ x_3) = (c + de)\overline{x}_2 + (b + dg)\overline{x}_3 + (a + eg)(x_2 \circ x_3)$.

Further simplification can be obtained by introducing a new node: $f_A(1, x_2, x_3) = c\overline{x}_2 + b\overline{x}_3 + a(x_2 \circ x_3) + (ex_2 + gx_3) \cdot (e\overline{x}_2 + g\overline{x}_3 + d)$ $f_A(1, x_2, x_3, x_4) = c\overline{x}_2 + b\overline{x}_3 + a(x_2 \circ x_3) + e(x_2 \circ x_4) + g(x_3 \circ x_4) + d\overline{x}_4$, which corresponds to the circuit of Fig. 1b.

Leningrad Electrotechnical Institute
named after V. I. Ulyanov (Lenin)

Received
26 VI 1956

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Note: Figure translations are in progress. See original paper for figures.

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