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Abstract

Full Text

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HYDROMECHANICS

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MOTION OF A PLATE OF INFINITE SPAN NEAR THE FREE SURFACE OF AN IDEAL WEIGHTLESS LIQUID

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The paper gives a solution of the problem of the motion of a plate of infinite span near the free surface of an ideal weightless liquid. The solution is carried out by the Kirchhoff method. The problem considers flows in which the upper surface of the plate is washed partly or completely by the liquid stream, as well as limiting cases in which separation of the free jets occurs directly from the leading edge of the plate.

In solving the problem, a plate of width b is placed so that its leading edge (point D) coincides with the origin of a rectangular coordinate system, the x-axis is directed along the plate DA, and the y-axis vertically upward. The velocity of the flow at infinity in front of the plate has a prescribed magnitude V_0 and makes an angle α with the x-axis (the angle of attack). From the upper surface of the plate (point B) a jet separates, whose thickness at infinity is equal to δ ($0 \leqslant \delta \leqslant \infty$); the angle at which the jet is established at infinity is equal to θ (Fig. 1).

Fig. 1. Physical plane of the flow

If a parametric variable $u = u_1 + iu_2$ is introduced into the consideration, then the complex coordinate of any point of the liquid, $z = x + iy$, is determined by the equality

$$\frac{\pi(1+\gamma^2)}{2\delta(\beta^2-\gamma^2)^2}z = \frac{A}{2}\ln\frac{(\varepsilon^2+\beta^2)(u+i\varepsilon)^2}{4\varepsilon^2(u^2-\beta^2)} - \frac{B}{2}\ln\frac{(\varepsilon^2+\gamma^2)(u^2-\beta^2)}{(\varepsilon^2+\beta^2)(u^2-\gamma^2)}$$
$$- i\frac{C}{2\gamma}\ln\frac{(\gamma-i\varepsilon)^2(\gamma+u)^2}{(\gamma^2+\varepsilon^2)(\gamma^2-u^2)} - i\frac{2\beta^2E+G}{4\beta^3}\ln\frac{(\beta-i\varepsilon)^2(\beta+u)^2}{(\beta^2+\varepsilon^2)(\beta^2-u^2)}$$
$$+ \frac{G}{2\beta^2}\left(\frac{iu}{u^2-\beta^2} - \frac{\varepsilon}{\varepsilon^2+\beta^2}\right) + \frac{F}{2}\frac{u^2+\varepsilon^2}{(u^2-\beta^2)(\varepsilon^2+\beta^2)} \qquad (u_1>0,\ u_2>0), \tag{1}$$

where ε, β, and γ are related to α and θ by the equalities

$$\varepsilon = \gamma\frac{1-\gamma\tan(\theta/2)}{\gamma+\tan(\theta/2)} = \beta\frac{1-\beta\tan(\alpha/2)}{\beta+\tan(\alpha/2)}; \tag{2}$$

$$A = \frac{2\varepsilon^2(1-\varepsilon)^2}{(\varepsilon^2+\gamma^2)(\varepsilon^2+\beta^2)^2};$$

$$B = -\frac{1+\gamma^2}{(\beta^2-\gamma^2)^2}\cos\theta; \quad C = -\frac{\gamma(1+\gamma^2)}{(\beta^2-\gamma^2)^2}\sin\theta;$$

$$E = \frac{2\varepsilon^3(1-\varepsilon)^2}{(\varepsilon^2+\gamma^2)(\varepsilon^2+\beta^2)^2} + \frac{(1+\gamma^2)\gamma}{(\beta^2-\gamma^2)^2}\sin\theta \quad (E = A\varepsilon - C); \tag{3}$$

$$F = \frac{1+\beta^2}{\beta^2-\gamma^2}\cos\alpha; \quad G = \frac{(1+\beta^2)\beta}{\beta^2-\gamma^2}\sin\alpha.$$

On the plate $u_1 = 0$ and $u = iu_2$, so that the coordinate x of the plate is determined by the equality

$$\frac{\pi(1+\gamma^2)}{2\delta(\beta^2-\gamma^2)^2}x = \frac{A}{2}\ln\frac{(\varepsilon^2+\beta^2)(u_2+\varepsilon)^2}{4\varepsilon^2(u_2^2+\beta^2)} -$$
$$-\frac{B}{2}\ln\frac{(\varepsilon^2+\gamma^2)(u_2^2+\beta^2)}{(\varepsilon^2+\beta^2)(u_2^2+\gamma^2)} - \frac{C}{\gamma}\left(\arctan\frac{u_2}{\gamma} - \arctan\frac{\varepsilon}{\gamma}\right) +$$
$$+\frac{2\beta^2E+G}{2\beta^3}\left(\arctan\frac{u_2}{\beta} - \arctan\frac{\varepsilon}{\beta}\right) +$$
$$+\frac{F}{2}\frac{u_2^2-\varepsilon^2}{(\varepsilon^2+\beta^2)(u_2^2+\beta^2)} + \frac{C}{2\beta^2}\frac{(u_2-\varepsilon)(\beta^2-\varepsilon u_2)}{(\varepsilon^2+\beta^2)(u_2^2+\beta^2)}. \tag{4}$$

Fig. 2. Fluid flows around the plate ($\varepsilon \neq 0$); $b = 1$; $a = 0.2$

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Fig. 3. Fluid flows around the plate ($\varepsilon = 0$); $b = 1$; $\alpha = 0.2$.

Figure 3: Fig. 3. Fluid flows around the plate ($\varepsilon = 0$); $b = 1$; $\alpha = 0.2$.

To determine the length of the plate b and the coordinate d of the point at which the upper jet separates from the plate, we shall have

$$\begin{aligned} \frac{\pi(1+\gamma^2)}{2\delta(\beta^2-\gamma^2)^2}\, b &= \frac{A}{2}\ln\frac{\varepsilon^2+\beta^2}{4\beta^2} - \frac{B}{2}\ln\frac{\varepsilon^2+\gamma^2}{\varepsilon^2+\beta^2} + \frac{C}{\gamma}\left(\frac{\pi}{2} - \arctan\frac{\varepsilon}{\gamma}\right) \\ &\quad + \frac{2\beta^2 E + G}{2\beta^3}\left(\frac{\pi}{2} - \arctan\frac{\varepsilon}{\beta}\right) + \frac{F}{2(\varepsilon^2+\beta^2)} - \frac{G}{2\beta^2}\frac{\varepsilon}{\varepsilon^2+\beta^2}, \\ \frac{\pi(1+\gamma^2)}{2\delta(\beta^2-\gamma^2)^2}\, d &= \frac{A}{2}\ln\frac{\varepsilon^2+\beta^2}{4\beta^2} - \frac{B}{2}\ln\frac{\beta^2(\varepsilon^2+\gamma^2)}{\gamma^2(\varepsilon^2+\beta^2)} - \frac{C}{\gamma}\arctan\frac{\varepsilon}{\gamma} \\ &\quad - \frac{2\beta^2 E + G}{2\beta^3}\arctan\frac{\varepsilon}{\beta} - \frac{\varepsilon(F\varepsilon + G)}{2\beta^3(\varepsilon^2+\beta^2)}. \end{aligned} \tag{5}$$

To determine the shape of the free jets it is necessary, in equality (1), to put $u = u_1$ ($u_2 = 0$); in this case for the jet BG, $0 \le u_1 \le \gamma$, for

Fig. 3. Fluid flows around the plate ($\varepsilon = 0$); $b = 1$; $\alpha = 0.2$.

the jet AE, $\beta \le u_1 \le \infty$, and on the free surface OC, $\gamma \le u_1 \le \beta$. The stream function is determined by the equality

$$\begin{aligned} \Psi = \frac{\delta V_0}{\pi}\Bigg[&\arccos\frac{u_1^2-u_2^2-\beta^2}{\sqrt{(u_1^2-u_2^2-\beta^2)^2+4u_1^2u_2^2}} \\ &- \arccos\frac{u_1^2-u_2^2-\gamma^2}{\sqrt{(u_1^2-u_2^2-\gamma^2)^2+4u_1^2u_2^2}}\Bigg] \\ &- \frac{\delta V_0}{\pi}(\beta^2-\gamma^2)\frac{1+\beta^2}{1+\gamma^2}\frac{2u_1u_2}{(u_1^2-u_2^2-\beta^2)^2+4u_1^2u_2^2} \qquad (u_1 \ge 0;\ u_2 \ge 0). \end{aligned} \tag{6}$$

The pressure at any point of the fluid is determined by the equality

$$\frac{2\Delta P}{\rho V_0^2} = 1 - \frac{[u_1^2+(u_2+\varepsilon)^2]\,[u_1^2+(u_2-1)^2]}{[u_1^2+(u_2-\varepsilon)^2]\,[u_1^2+(u_2+1)^2]}, \tag{7}$$

and the pressure at any point of the plate is

Fig. 4. Pressure distributions over the plate.

Figure 4: Fig. 4. Pressure distributions over the plate.

$$\frac{2\Delta P_{\mathrm{p}}}{\rho V_0^2} = 1 - \frac{(u_2+\varepsilon)^2(u_2-1)^2}{(u_2-\varepsilon)^2(u_2+1)^2}. \tag{8}$$

The lift and drag coefficients, referred to the dynamic pressure $\rho V_0^2/2$ and to the chord of the plate b with unit span, are determined by the formulas

$$\begin{aligned} C_y &= \frac{2\delta}{b}\,\frac{(\beta^2-\gamma^2)^2}{1+\gamma^2}\left[\left(\frac{C}{\gamma}+\frac{E}{\beta}+\frac{G}{2\beta^3}\right)\cos\alpha + A\sin\alpha\right], \\ C_x &= \frac{2\delta}{b}\,\frac{(\beta^2-\gamma^2)^2}{1+\gamma^2}\left[\left(\frac{C}{\gamma}+\frac{E}{\beta}+\frac{G}{2\beta^3}\right)\sin\alpha - A\cos\alpha\right]. \end{aligned} \tag{9}$$

From the second formula (5) we note that $\varepsilon = 0$ corresponds to the limiting case in which separation of the upper jet occurs directly

Fig. 4. Pressure distributions over the plate. $\alpha = 0.2$; $\varepsilon = \gamma^2$; $\bar{P} = \frac{2(P-P_0)}{\rho V_0^2}$ is the pressure coefficient; x/b is the distance from the leading edge in fractions of the chord b

from the leading edge of the plate ($d = 0$). The length of the plate in this case will be calculated by the formula

$$\frac{b\pi}{\delta} = \frac{2\cos\alpha(\cos\alpha-\cos\theta)}{\sin^2\alpha} - 2\cos\theta\ln\frac{\tan\frac{\theta}{2}}{\tan\frac{\alpha}{2}} + \pi\,\frac{1-\cos(\theta-\alpha)}{\sin\alpha}, \tag{10}$$

and C_y and C_x, respectively, by the formulas

$$C_y = 2\frac{\delta}{b}\,\frac{1-\cos(\theta-\alpha)}{\tan\alpha}, \qquad C_x = 2\frac{\delta}{b}\,[1-\cos(\theta-\alpha)]. \tag{11}$$

Figure 2 shows the fluid flows around the plate for different thicknesses of the upper jet being shed and different ε; Fig. 3 shows flows in which separation of the upper jet occurs directly from the leading edge of the plate ($\varepsilon = 0$). Figure 4 gives the pressure distributions over the upper and lower surfaces of the plate.

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Note: Figure translations are in progress. See original paper for figures.