# HYDROMECHANICS

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**Abstract**

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## THE PROBLEM OF A POINT EXPLOSION

*(Presented by Academician L. I. Sedov, 23 VII 1956)*

Let an instantaneous release of energy $E_0$—an explosion—take place at some point in a quiescent homogeneous gas. The intense shock wave arising in the explosion gradually weakens as it recedes from the center of the explosion. It is required to determine the motion of the gas behind the shock wave. (The problem is formulated analogously for flows with plane and cylindrical waves.)

As long as the initial pressure in the gas may be considered negligibly small in comparison with the pressure behind the shock front, the motion is self-similar (a strong explosion); the corresponding problem has an exact analytic solution ([1]). As the shock wave weakens, neglect of the initial pressure—the "counter-pressure" —becomes inadmissible. The problem of a point explosion with allowance for counter-pressure has been solved, in an approximate formulation, by considering motions close to the self-similar one ([2,3]), and, in the exact formulation, by the method of grids with the use of high-speed computing machines for the calculations ([4,5]). Below a solution is given for the problem of a point explosion by the method of expanding the solution in series in powers of

$$\varepsilon = \frac{\gamma - 1}{\gamma + 1},$$

where $\gamma$ is the ratio of specific heats ([6]).

We shall write the equations of one-dimensional motion of a gas in the form

$$\frac{\partial R}{\partial m} = \frac{1}{\rho R^{\nu-1}}, \qquad \frac{\partial^2 R}{\partial t^2} = -R^{\nu-1}\frac{\partial p}{\partial m}, \qquad \frac{\partial}{\partial t}\frac{p}{\rho^\gamma} = 0. \tag{1}$$

Here $R$ is the distance of a particle from the center (axis, plane) of symmetry; $p$ is the pressure; $\rho$ is the density. The independent variables are the time $t$ and the Lagrangian variable $m = \rho_1^0 r^\nu/\nu$, where $r$ is the initial distance of the particle from the center (axis, plane) of symmetry; $\rho_1^0$ is the initial density of the gas; $\nu = 1, 2, 3$ for flows, respectively, with plane, cylindrical, and spherical waves.

The expressions for the flow parameters behind the shock wave (with the asterisk index) may be written in the form:

$$p^* = \frac{2}{\gamma+1}\rho_1^0 \dot{R}_0^2 - \frac{\gamma-1}{\gamma+1}p_1^0, \qquad \rho^* = \frac{\dfrac{\gamma+1}{\gamma-1}\rho_1^0}{1+\dfrac{2}{\gamma-1}\dfrac{a_1^2}{\dot{R}_0^2}}, \qquad m^* = \frac{\rho_1^0 R_0^\nu}{\nu}. \qquad (2)$$

Here $R_0(t)$ is the law of motion of the shock wave; $p_1^0$ and $a_1$ are the pressure and the speed of sound in the undisturbed gas.

We shall seek the solution of system (1) in the form

$$R = R_0 + \varepsilon R_1 + \cdots, \qquad p = p_0 + \varepsilon p_1 + \cdots, \qquad \rho = \frac{\rho_0}{\varepsilon} + \rho_1 + \cdots. \qquad (3)$$

Then, using the boundary conditions (2) and carrying out the corresponding calculations, we find

$$p_0 = -\frac{2}{\gamma+1}\rho_1^0 \dot{R}_0^2 + \rho_1^0 \frac{R_0 \ddot{R}_0}{\nu} - \frac{\ddot{R}_0}{R_0^{\nu-1}} m;$$

$$\rho_0 = \frac{\rho_1^0}{1+\dfrac{2}{\gamma-1}\dfrac{a_1^2}{\dot{R}_0^2(t_1)}} \frac{p_0^{1/\gamma}}{\left[\dfrac{2}{\gamma+1}\rho_1^0 \dot{R}_0^2(t_1)\right]^{1/\gamma}};$$

$$R_1 = -\frac{1}{R_0^{\nu-1}} \int_{t_1}^{t} \frac{\left[1+\dfrac{2}{\gamma-1}\dfrac{a_1^2}{\dot{R}_0^2(t_1)}\right] R_0^{\nu-1}(t_1) \dot{R}_0^{2/\gamma+1}(t_1)\, dt_1}{\left[\dot{R}_0^2 + \dfrac{\gamma+1}{2}\dfrac{R_0\ddot{R}_0}{\nu}\left(1-\dfrac{R_0^\nu(t_1)}{R_0^\nu}\right)\right]^{1/\gamma}}; \qquad (4)$$

$$p_1 = (\gamma-1)\frac{\ddot{R}_0}{R_0^\nu}\int_{m^*}^{m} R_1\, dm - \frac{1}{R_0^{\nu-1}}\int_{m^*}^{m} \frac{\partial^2 R_1}{\partial t^2}\, dm - p_1^0;$$

$$\rho_1 = \frac{\rho_0}{\gamma}\left[\frac{p_1}{p_0} + \frac{\gamma+1}{2\gamma}\frac{a_1^2}{\dot{R}_0^2(t_1)}\right].$$

The variable $t_1$ is connected with $m$ by the relation $m = \rho_1^0 R_0^\nu(t_1)/\nu$, whence it follows that $t_1(m)$ is the time at which the shock wave passes through the particle with Lagrangian coordinate $m$.

The formulas (4) give, in the approximation under consideration, the solution of the Cauchy problem for the equations of one-dimensional unsteady gas motion with initial data on the shock wave.

To determine the function $R_0(t)$ in the explosion problem, we use the law of conservation of energy. The total (kinetic and internal) energy of the moving gas must be equal to the sum of the energy $E_0$ released in the explosion and the initial energy of the gas:

$$\int_0^{m^*}\left[\frac{1}{2}\left(\frac{\partial R}{\partial t}\right)^2+\frac{1}{\gamma-1}\frac{p}{\rho}\right]dm=E+\int_0^{m^*}\frac{1}{\gamma-1}\frac{p_1^0}{\rho_1^0}\,dm. \tag{5}$$

Here $2[(\nu-1)\pi+\delta_{1\nu}]E=E_0$ ($\delta_{\nu 1}=1$ for $\nu=1$; $\delta_{\nu 1}=0$ for $\nu\neq 1$; for $\nu=2$ and $\nu=1$, $E_0$ is the energy referred, respectively, to unit length and to unit area of the charge).

Substituting the expansions (3) into the energy equation (5), we reduce it to the form

$$\left[\frac{1}{2}\dot{R}_0^2-\frac{a_1^2}{\gamma(\gamma-1)}\right]m^*+\int_0^{m^*}\left\{\frac{\gamma-1}{\gamma+1}\dot{R}_0\frac{\partial R_1}{\partial t}+\frac{1}{\gamma+1}\frac{p_0}{\rho_0}\left[1+\frac{\gamma-1}{\gamma+1}\left(\frac{p_1}{p_0}-\frac{\rho_1}{\rho_0}\right)\right]\right\}dm=E+O(\varepsilon^2).$$

We introduce, for measuring length and time respectively, the scales

$$L=\left(\frac{E}{\gamma p_1^0}\right)^{1/\nu}$$

and $L/a_1$, and set $\dot{R}_0^2=\varphi$ and $R_0^\nu/\gamma=x$ (the quantity $\varphi$ is connected with the variable $q$ of works [1, 2] by the relation $\varphi q=1$). Then the equation written above is transformed into the form

$$\begin{gathered}\left[\frac{1}{2}\varphi(x)-\frac{1}{\gamma(\gamma-1)}\right]x+\frac{2}{(\gamma+1)^2}\int_0^x\{\varphi(x)+\\+\frac{\gamma+1}{4}\varphi'(x)(x-s)\Big\}^{(\gamma-1)/\gamma}\left[1+\frac{2}{\gamma-1}\frac{1}{\varphi(s)}\right]\left[1-\frac{\gamma-1}{2\gamma}\frac{1}{\varphi(s)}\right]\varphi^{1/\gamma}(s)\,ds\\-\frac{\gamma-1}{\gamma+1}x^{(\gamma-1)/\gamma}\varphi(x)\frac{d}{dx}\left\{\frac{1}{x^{(\gamma-1)/\gamma}}\int_0^x\int_s^x\frac{\left[1+\frac{2}{\gamma-1}\frac{1}{\varphi(s)}\right]\varphi^{1/\gamma}(s)\,ds\,ds}{\left[\varphi(x)+\frac{\gamma+1}{4}\varphi'(x)(x-s)\right]^{1/\gamma}}\right\}=1.\end{gathered} \tag{6}$$

Thus the problem of a point explosion in the approximation under consideration has been reduced to the problem of finding, from this integro-differential equation, the function $\varphi(x)$, depending on one variable.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

We shall confine ourselves to considering the initial period of motion, i.e., small values of $x$, and shall seek a solution of equation (6) in the form

$$\varphi(x) = \frac{1}{\alpha(\gamma,\nu)x} + A(\gamma,\nu) + ... \quad (7)$$

Fig. 1

Let us note that retaining the first term in this series corresponds to the solution of the problem of automodel motion for a strong explosion, while retaining the first two terms corresponds to the solution of the explosion problem with allowance for counterpressure in the initial stage of motion ([2,3]).

For automodel motion, integrating the equation

$$\dot{R}_0^2 = \frac{\nu}{\alpha R_0^\nu},$$

and returning to dimensional variables, we find the law of propagation of the shock wave in the form

$$R_0 = \left(\frac{E_0}{\alpha\rho_1^0}\right)^{1/(\nu+2)} t^{2/(\nu+2)}. \quad (8)$$

Here

$$\alpha = \frac{2\left[(\nu-1)\pi + \delta_{1\nu}\right]}{\nu\left(\frac{2+\nu}{2}\right)^2}\,\tilde{\alpha}.$$

Fig. 2

In Fig. 1 the computed values of $\alpha$ as a function of $\gamma$ for $\nu = 1, 2, 3$ are plotted with solid lines, and the exact values of $\alpha$ ([1]) with dashed lines. The approximate values at $\gamma = 1.4$ agree very well with the exact ones.

In Fig. 2 the computed values of $A(\gamma,\nu)$ as a function of $\gamma$ are presented for $\nu = 1, 2, 3$. According to the linear theory of allowing for counterpressure, the value of $A$ at $\gamma = 1.4$ is equal to 1.91 for $\nu = 3$ ([2]), and to 1.97 and 2.14, respectively, for $\nu = 2$ and $\nu = 1$ ([7]). Such a discrepancy can be explained as follows. The difference between the values of pressure and density behind the discontinuity when the initial pressure is taken into account and in automodel motion,

characterized by the quantity $A$, becomes noticeable only when the shock is greatly weakened and when, consequently, no sufficiently strong compression of the gas occurs in the shock. Thus, the influence of counterpressure begins to make itself felt only in that phase of propagation of the shock wave in which the use of the theory being set forth already becomes poorly justified.

**Fig. 3.** Dependence of the pressure behind the front of the blast wave on the distance to the center of the explosion according to various theories.

$$L = \sqrt[3]{\frac{E_0}{\gamma p_1^0}}, \qquad \gamma = 1.4$$

In Fig. 3, for the case of spherical symmetry, values of $p^*/p_1^0$ are given as a function of the distance to the center of the explosion, calculated according to various theories: curve $I$—under the assumption of self-similarity of the motion ([1]), $II$—for the linearized solution of the explosion problem with counterpressure ([2]), $III$—for the solution of the problem in the exact formulation by the mesh method ([4]), the dashed curve—for the approximate solution of equation (6) with retention of the first two terms in series (7). The last solution agrees well with the exact one up to values $p^*/p_1^0 = 1.5$.

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*Note: Figure translations are in progress. See original paper for figures.*