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Abstract

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APPROXIMATE DISPERSION RELATIONS FOR NUCLEON-NUCLEON SCATTERING

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Dispersion relations for nucleon-nucleon scattering have been considered by several authors ([1–3]). The relations obtained by them could not be compared with experiment, since they contained unobservable quantities, as well as unknown quantities associated with the scattering of antinucleons by nucleons.

In the present work approximate dispersion relations are investigated. The scattering of nucleons by nucleons in the absence of antinucleons is considered. We start from the dispersion relation obtained in the usual way ([4]),

$$D(E) - \frac{1}{2}\left(1 + \frac{E}{M}\right) D(M) - \frac{1}{2}\left(1 - \frac{E}{M}\right) D(-M) =$$

$$= \frac{E^2 - M^2}{\pi} P \int_0^\infty \frac{A(E')\,dE'}{(E'-E)({E'}^2 - M^2)} - \frac{E^2 - M^2}{\pi} P \int_0^\infty \frac{A(-E')\,dE'}{(E'+E)({E'}^2 - M^2)}. \tag{1}$$

Here D and A are the real and imaginary parts of the nucleon-nucleon scattering amplitude; E is the energy of the scattered nucleon in the coordinate system in which the sum of the momenta of the scatterer before and after the collision is zero ($\mathbf{p} + \mathbf{p}' = 0$). The amplitudes for scattering of a nucleon by a nucleon, $D(-M)$ and $A(-E')$, are expressed linearly in terms of the amplitudes for scattering of an antinucleon by a nucleon with positive energy. Thus the dispersion relation (1) connects the amplitudes of nucleon-nucleon and antinucleon-nucleon scattering.

In the approximation under consideration, (1) has the form

$$D(E) - \frac{1}{2}\left(1 + \frac{E}{M}\right) D(M) =$$

$$= \frac{E^2 - M^2}{\pi} P \int_{\frac{M^2-p^2}{\sqrt{M^2+p^2}}}^\infty \frac{A(E')\,dE'}{(E'-E)({E'}^2 - M^2)} + C\,\frac{(E^2-M^2)\delta_{0T}\delta_{1S}}{E - \dfrac{M^2 - p^2 - 2M\varepsilon}{\sqrt{M^2+p^2}}}, \tag{2}$$

where the last term is the contribution of the deuteron intermediate state; δ_{0T} and δ_{1S} are Kronecker symbols, different from zero for values of the total isotopic spin $T = 0$ and total ordinary spin $S = 1$; C is an energy-independent constant which must be determined from comparison with experiment; ε is the binding energy of the deuteron.

Such an approximation is satisfactory in that energy region where the magnitude of the integral is determined mainly by the behavior of the integrand for $E' \sim E$. For this, the function $A(E')$ under the integral in this energy region must possess a large

derivative, for the integration is carried out in the sense of the principal value. The scattering amplitude $A(E')$ appearing under the integral in (2) is not an observable quantity in the region

$$\sqrt{M^2 + \mathbf{p}^2} > E' \geq \frac{M^2 - \mathbf{p}^2}{\sqrt{M^2 + \mathbf{p}^2}}. \tag{3}$$

For forward scattering $\mathbf{p}^2 = 0$, and the unobservable region disappears. In nucleon–nucleon scattering it is necessary to take account of the identity of the particles and to consider not the scattering amplitude $f(\theta)$ through a certain angle θ, but a linear combination of the form

$$f(\theta) \pm f(\pi - \theta), \tag{4}$$

where θ is the scattering angle in the center-of-mass system. For this amplitude the unobservable region (3) remains for all θ. In order to carry out the symmetrization, let us write (2) in the center-of-mass system:

$$D(W, \mathbf{p}^2) - \frac{1}{2}\left(1 + \frac{W^2 - 2M^2 - 2\mathbf{p}^2}{2M\sqrt{M^2 + \mathbf{p}^2}}\right) D(2M, \mathbf{p}^2) =$$

$$= \frac{W^4 + 4(\mathbf{p}^2 - W^2)(M^2 + \mathbf{p}^2)}{\pi} P \int_{2M}^{\infty} \frac{2W' A(W', \mathbf{p}^2)\, dW'}{(W'^2 - W^2)\left[W'^4 + 4(\mathbf{p}^2 - W'^2)(W^2 + \mathbf{p}^2)\right]} +$$

$$+ C' \frac{W^4 - 4(W^2 - \mathbf{p}^2)(M^2 + \mathbf{p}^2)}{W^2 - 4M(M - \varepsilon)}, \tag{5}$$

where W is the total energy of the two-nucleon system, $C' = \dfrac{c}{2\sqrt{M^2 + \mathbf{p}^2}}$ and

$$\mathbf{p}^2 = \frac{W^2 - 4M^2}{8}(1 - \cos\theta).$$

The unobservable region in relation (5) lies within $2M \leq W' < 2\sqrt{M^2 + \mathbf{p}^2}$. In this region $\cos\theta' < -1$. The transition $\theta \to \pi - \theta$ is equivalent in (5) to the replacement

$$\mathbf{p}^2 \to \mathbf{p}'^2 = \frac{W^2 - 4M^2}{4} - \mathbf{p}^2.$$

For carrying out the symmetrization it is essential that

$$\frac{W^4 + 4(\mathbf{p}^2 - W^2)(\mathbf{p}^2 + M^2)}{W^4 + 4(\mathbf{p}^2 - {W'}^2)(\mathbf{p}^2 + M^2)} - \frac{W^4 + 4(\mathbf{p}'^2 - W^2)(\mathbf{p}'^2 + M^2)}{W^4 + 4(\mathbf{p}'^2 - {W'}^2)(\mathbf{p}'^2 + M^2)} \sim {W'}^2 - W^2. \tag{6}$$

Moreover, the $\cos\theta'$ and $\cos(\pi-\theta)'$ entering (5) under the integral are equal to:

$$\cos\theta' = 1 - \frac{8\mathbf{p}^2}{{W'}^2 - 4M^2};$$

$$\cos(\pi - \theta)' = -1 + \frac{8\mathbf{p}^2}{{W'}^2 - 4M^2} + 2\,\frac{{W'}^2 - W^2}{{W'}^2 - 4M^2}. \tag{7}$$

Therefore, if, in accordance with the assumption made earlier, the function $A(W')$ in the region $W' \sim W$ changes sufficiently rapidly, then the integral containing the difference (6) may be neglected in comparison with the integral containing the analogous sum, and in expression (7) for $\cos(\pi-\theta)'$ the last term may be neglected. Then

$$\cos(\pi - \theta)' = \cos(\pi - \theta') = -\cos\theta',$$

and carrying out operation (4) proves possible. In this approximation, for forward scattering ($\mathbf{p}^2 = 0$) the unobservable region completely disap-

and we obtain:

$$D(k) - \frac{1}{2}\left(\frac{3}{2} + \frac{k^2}{M^2} + \frac{\sqrt{k^2 + M^2}}{2M}\right) D(0) =$$

$$= \frac{2k^2(k^2 + M^2)}{\pi}\, P\int_0^\infty \frac{A(k')\,dk'}{k'({k'}^2 - k^2)({k'}^2 + M^2)} + C'\,\frac{3k^2(k^2 + M^2)}{k^2 + M\varepsilon}, \tag{8}$$

where

$$k^2 = \frac{W^2 - 4M^2}{4}.$$

An experimental verification of the dispersion relation (8) written above is of interest. The existing experimental data on the total cross section $\sigma(k)$ and on the angular distribution make it possible to calculate the integral of the total cross section and to compare the $D(k)$ obtained in this way with the experimentally measured one. This check will answer the question of the limits of applicability of the dispersion relation (8). In the region of very small energies (up to 6 MeV), where the scattering is well described by the S-wave, one may use the expansion

$$k \operatorname{ctg} \delta = -\frac{1}{a} + \frac{1}{2} r k^2 \tag{9}$$

and carry out a check of the dispersion relation (8). In this case it takes the form

$$\frac{1}{k} \sin 2\delta_{0,1}(k) + \left(\frac{3}{2} + \frac{k^2}{M^2} + \frac{\sqrt{k^2+M^2}}{2M} \right) a_{0,1} =$$

$$= \frac{4k^2(k^2+M^2)}{\pi} P \int_0^\infty \frac{\sin^2 \delta_{0,1}(k')\, dk'}{k'^2(k'^2-k^2)(k'^2+M^2)} + C' \frac{3k^2(k^2+M^2)}{k^2+M\varepsilon}, \tag{10}$$

where

$$a_{0,1} = \lim_{k\to 0} \left[\frac{1}{k} \sin 2\delta_{0,1}(k) \right],$$

and δ_0 and δ_1 are the phase shifts of singlet and triplet S-scattering, respectively. Substituting the expansion (9) into (10), differentiating with respect to k^2, and putting $k^2 = 0$, we obtain a relation containing r and a. These quantities are well known and are equal to [5]: $r_0 = 3 \cdot 10^{-13}$ cm, $r_1 = 1.704 \cdot 10^{-13}$ cm, $a_0 = -23.69 \cdot 10^{-13}$ cm, $a_1 = 5.38 \cdot 10^{-13}$ cm.

For singlet scattering the deuteron term is absent, and the left-hand side of (10) coincides with the right-hand side to an accuracy of 0.01%. The problem of determining r_0 from a_0 in this case cannot be solved. This is connected with the fact that r_0 is very sensitive to changes in the total cross section $\sigma(k)$. Substitution of the expansion (9) in place of the total cross section is only a rough check of the dispersion relations.

For triplet scattering, if the deuteron term is not taken into account, the left-hand side of (10) coincides with the right-hand side to an accuracy of 3%.

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Note: Figure translations are in progress. See original paper for figures.